Complex Networks meet Sparsity

Solving Problems Definable in First-Order Logic

Jan Dreier, Philipp Kuinke, Peter Rossmanith

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RWTH Aachen University, TU Wien

Algorithmic Meta-Theorems

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"All problems expressible in a certain logic
can be solved efficiently on certain graph classes"

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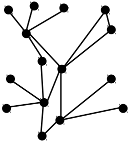
MSO on treewidth



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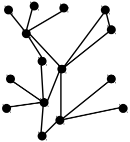

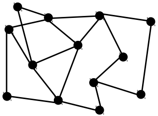

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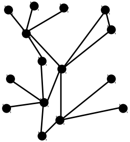
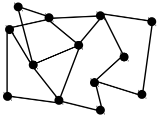
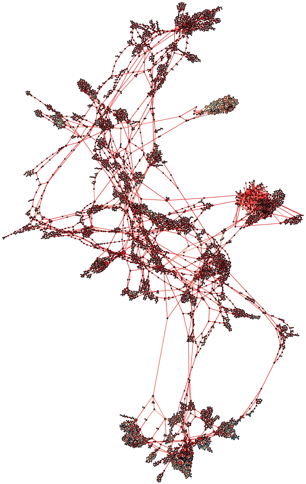
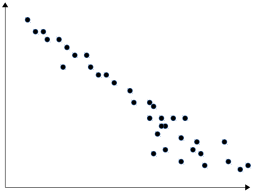


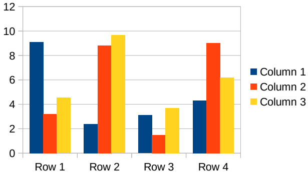


$$\mathcal{C} = \{G_1, \ldots, G_\infty\}$$

Network Science







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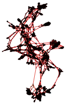

real world
networks

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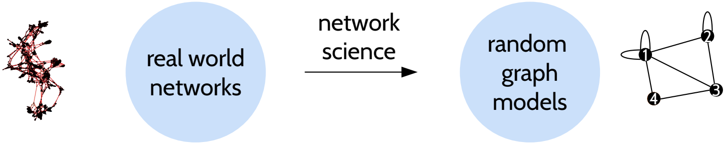
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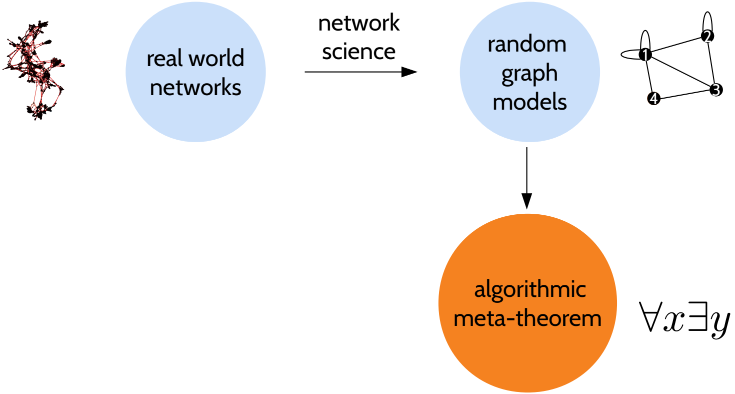

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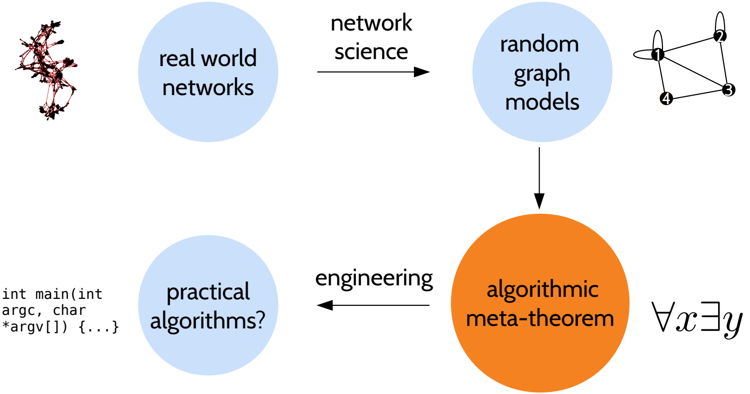
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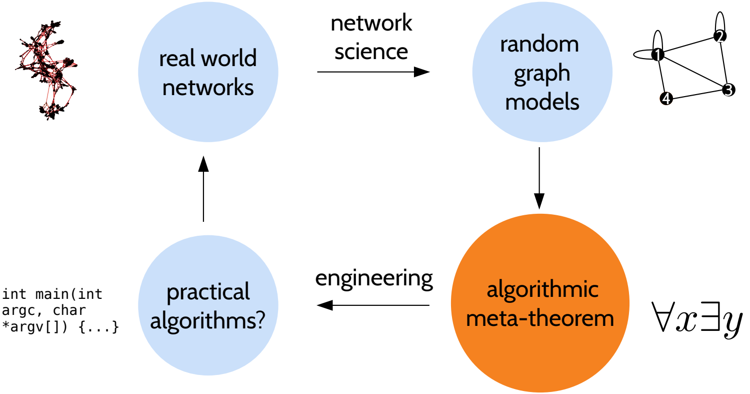

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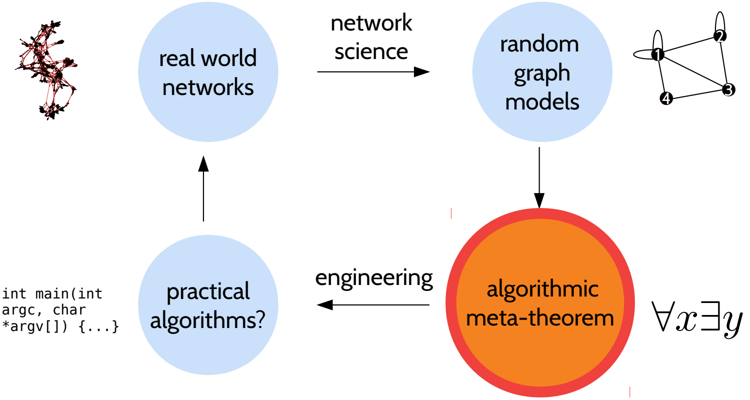

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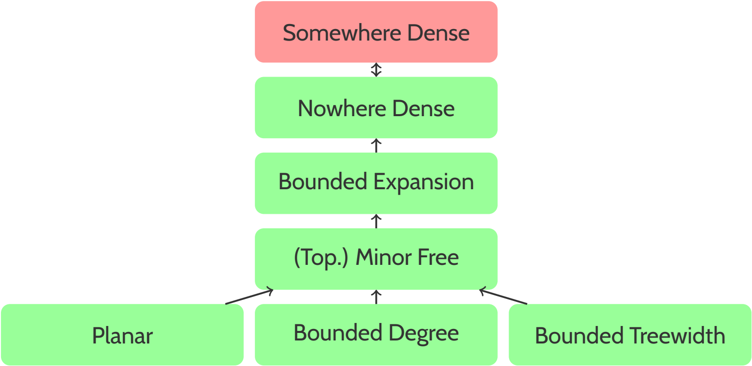
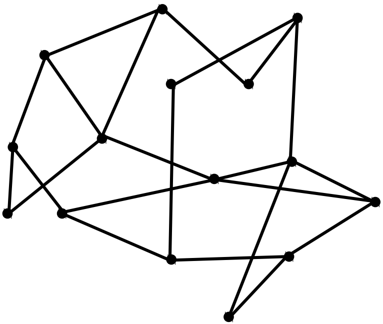
Sparse Graph Classes



What Are Sparse Graphs?



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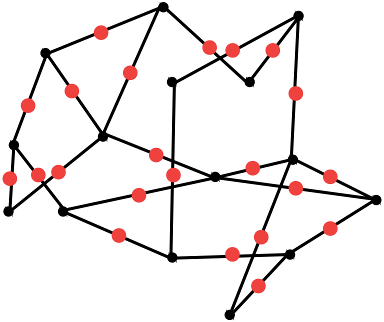

$$\frac{|E|}{|V|} \leq 2$$

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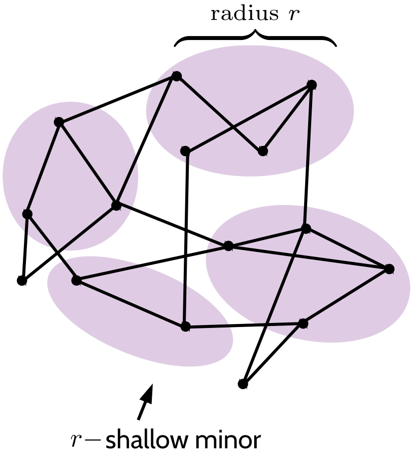

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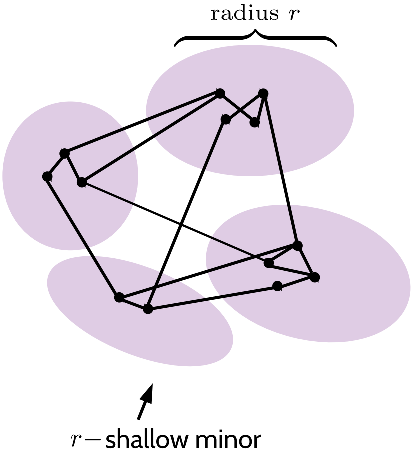

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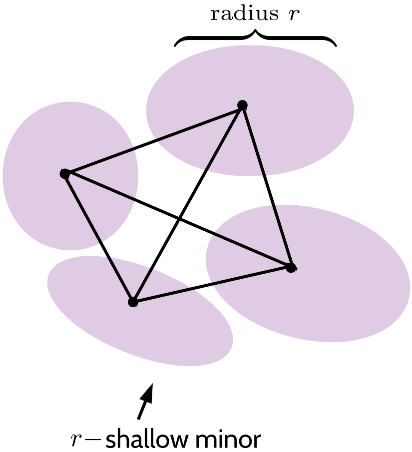

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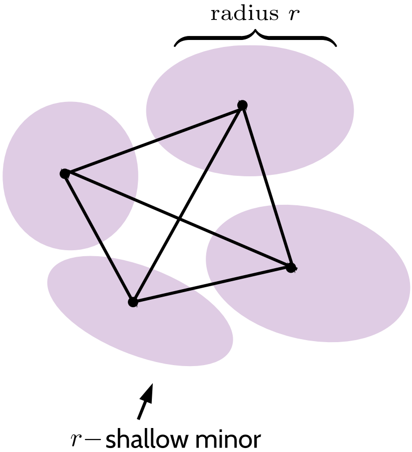

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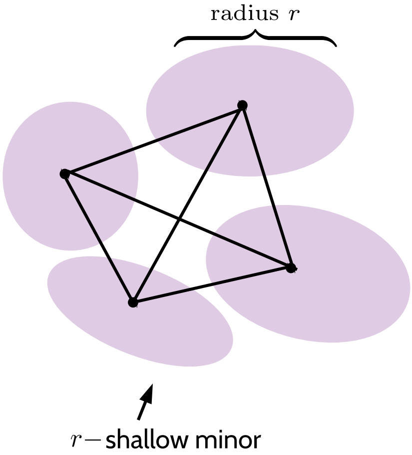

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real world: for small r (such as 1,2,3) sparse (w.r.t. gen. col. numbers)

[Nadara, Pilipczuk, Rabinovich, Reidl, Siebertz 2019]

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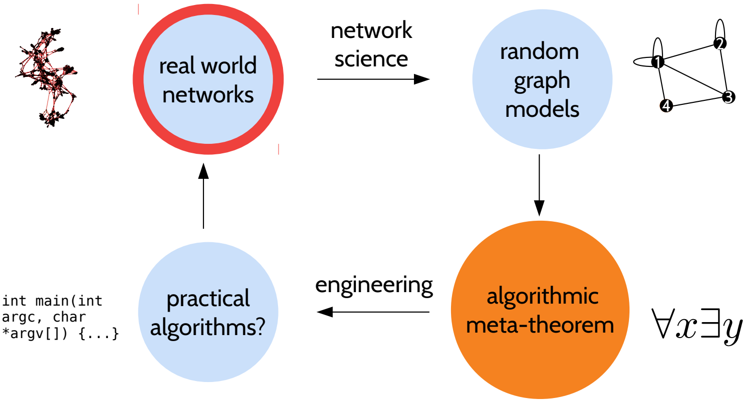

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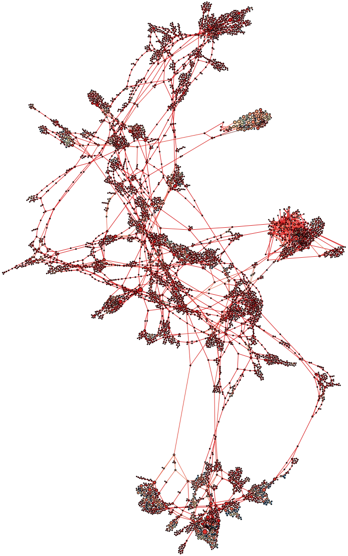

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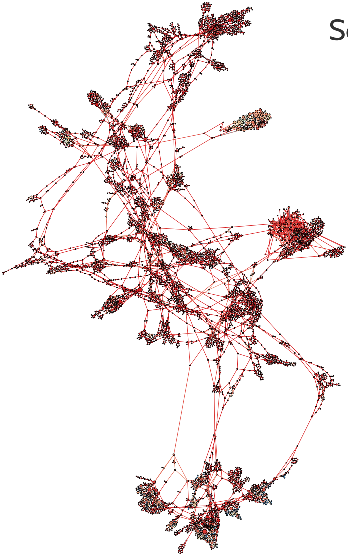

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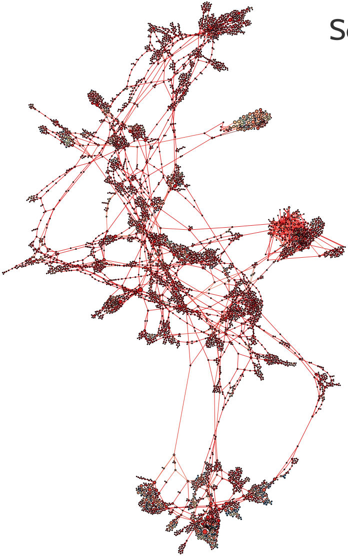

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  Fraction of vertices with degree k proportional to $k^{-\alpha}$ with $2 \leq \alpha \leq 3$?

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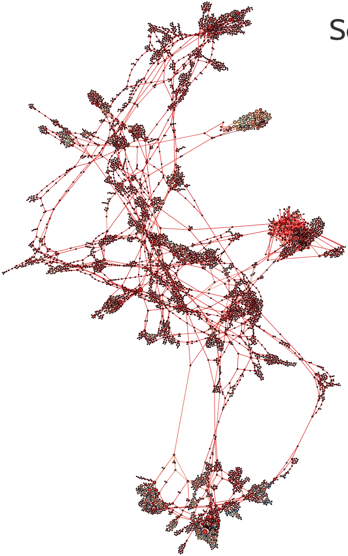

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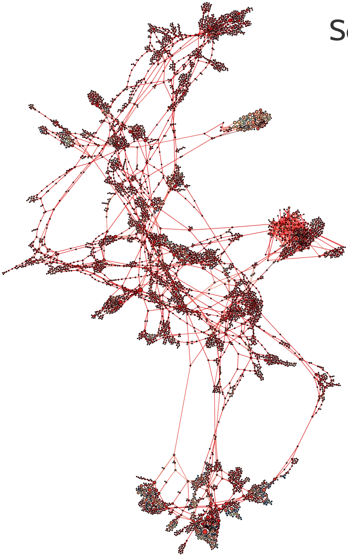

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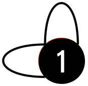
- **Small-world property**
  Everyone is close to everyone

Example: Preferential Attachment Model

Introduced by Barabási and Albert in 1999 to explain the structure of the world wide web.



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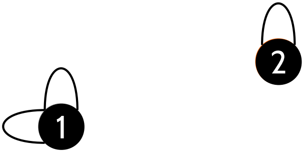

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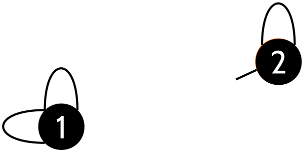

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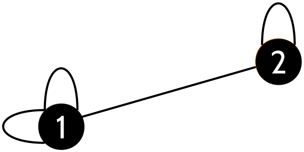

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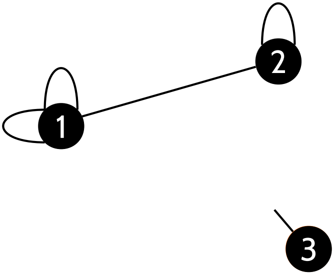

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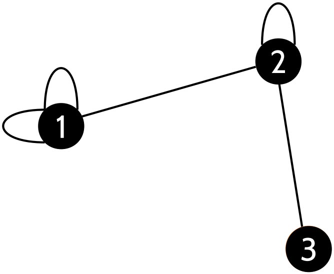

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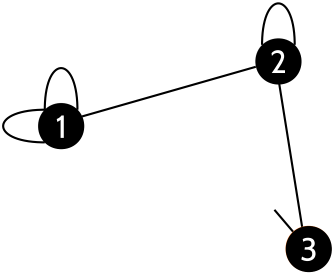

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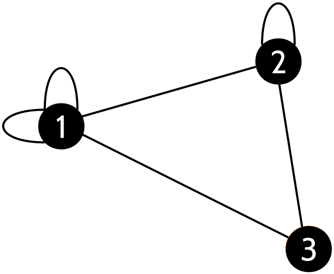

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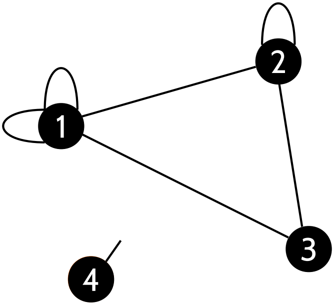

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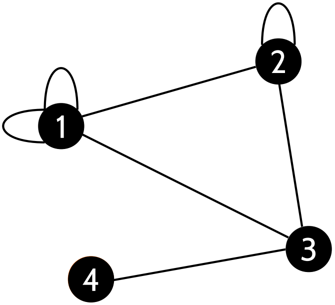

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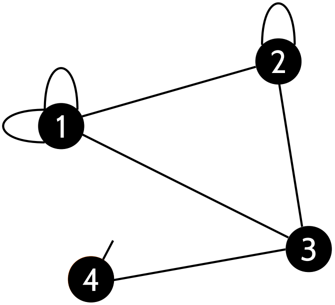

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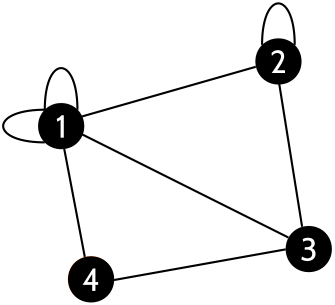

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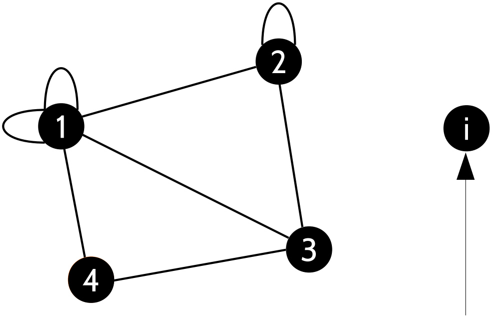
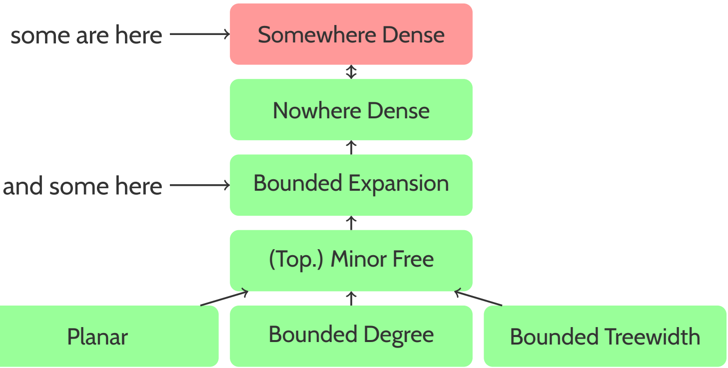


$$\text{expected degree} \approx \sqrt{\frac{n}{i}}$$

Sparsity of Random Graph Models



[Grohe 2001], [Farrell et. al. 2015], [Demaine et. al. 2019], [Dreier et. al. 2020]

Results

Our Result

A random graph model is *3-power-law-bounded* if (roughly speaking):

[Dreier, Kuinke, Rossmanith 2020]

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Theorem

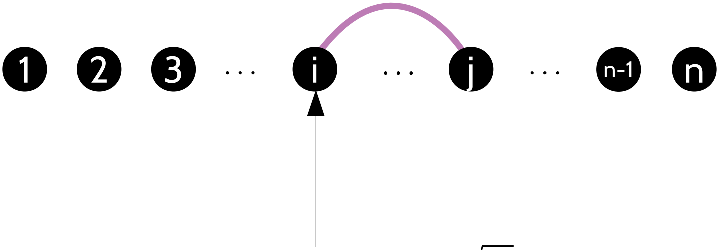
Given a first-order sentence φ and a graph G sampled from a 3-power-law-bounded model, one can decide whether φ is true on G in expected time $f(|\varphi|)n^{1+\varepsilon}$ for every $\varepsilon > 0$.

[Dreier, Kuinke, Rossmanith 2020]

Example: Chung–Lu Model

A more direct way to get a desirable degree distribution.

$$\frac{1}{\sqrt{i \cdot j}}$$



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α-power-law-boundedness

A random graph model with vertices $1, \ldots, n$ is 3-*power-law-bounded* if the probability that some subset of edges $E \subseteq \binom{\{1,\ldots,n\}}{2}$ is present is at most

$$\log(n)^{O(|E|^2)} \prod_{ij \in E} \frac{1}{\sqrt{i \cdot j}}.$$

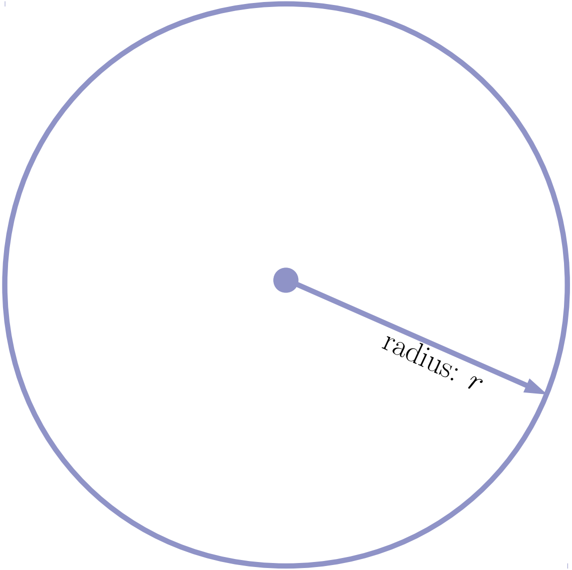
✓

- Preferential attachment model
- Chung–Lu model
- Erdös–Rényi model
- Configuration model
- …

✗

- Hyperbolic random graph model
- random intersection model
- Watts–Strogatz model
- Kleinberg model
- …

Asymptotic Structure of 3-power-law-bounded models


radius: r

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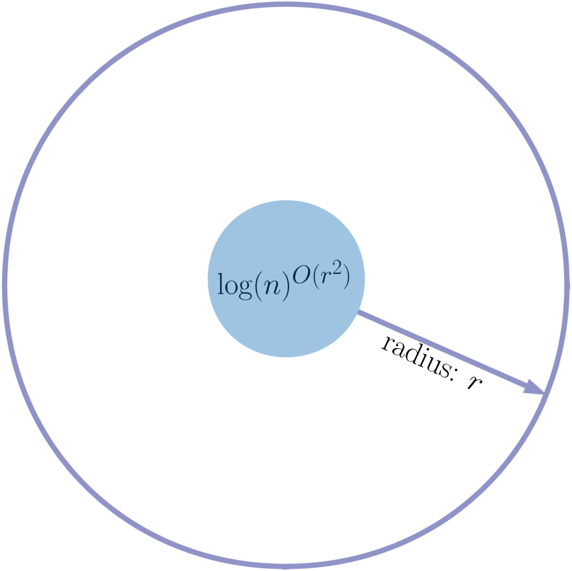

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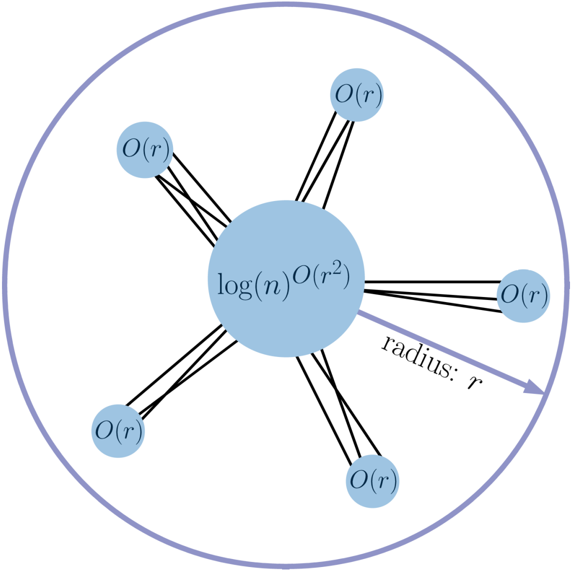

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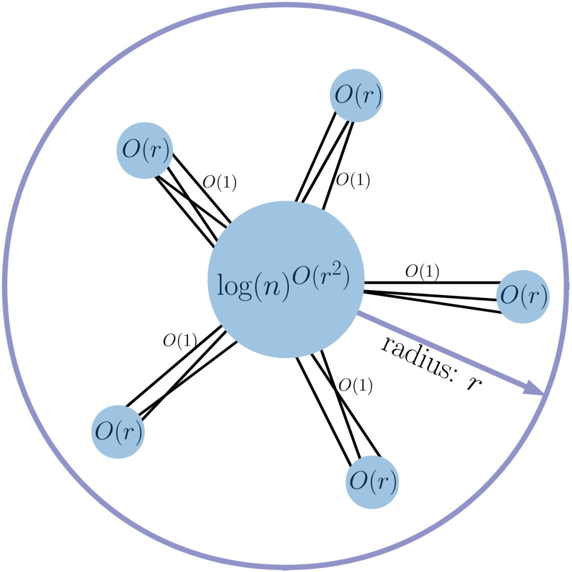

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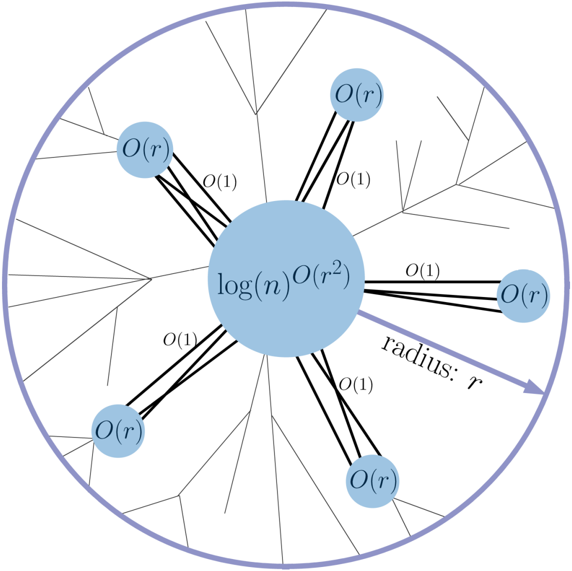
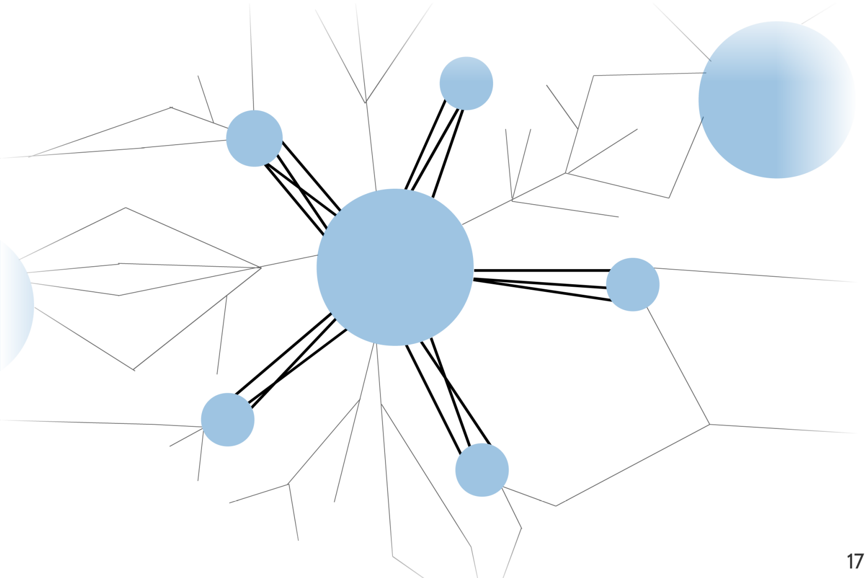


Algorithm

Input: graph sampled from 3-power-law-bounded model

Algorithm



Algorithm

Gaifman's theorem:
consider only
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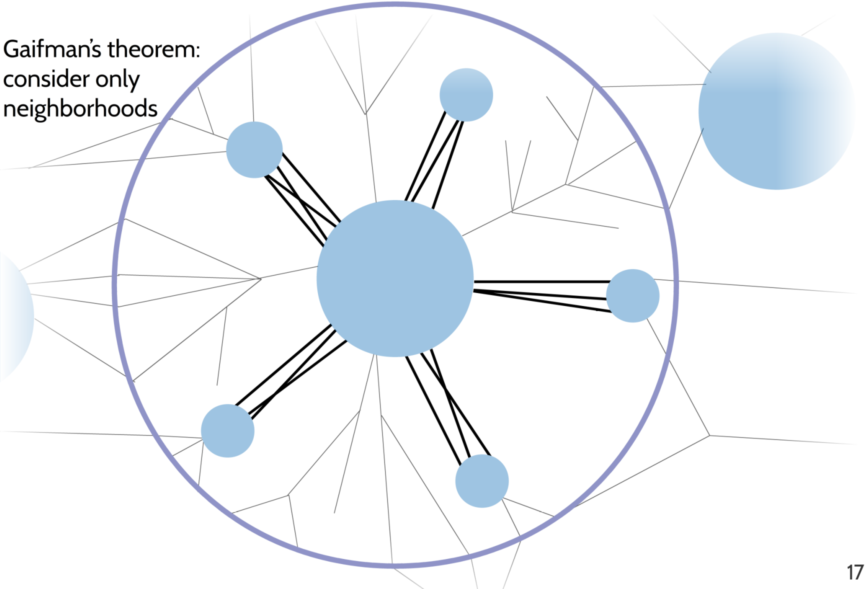

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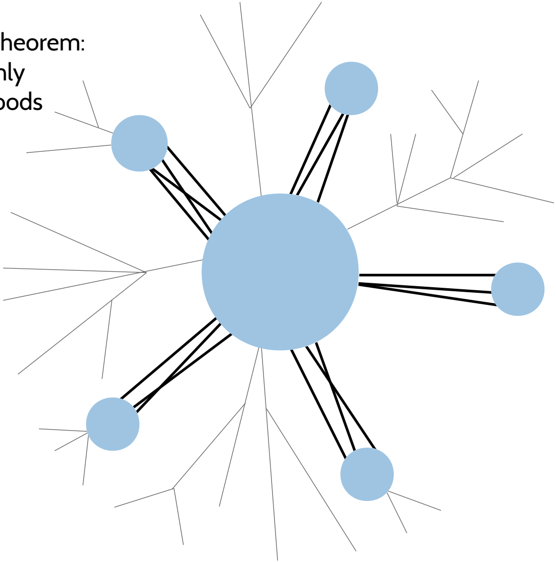
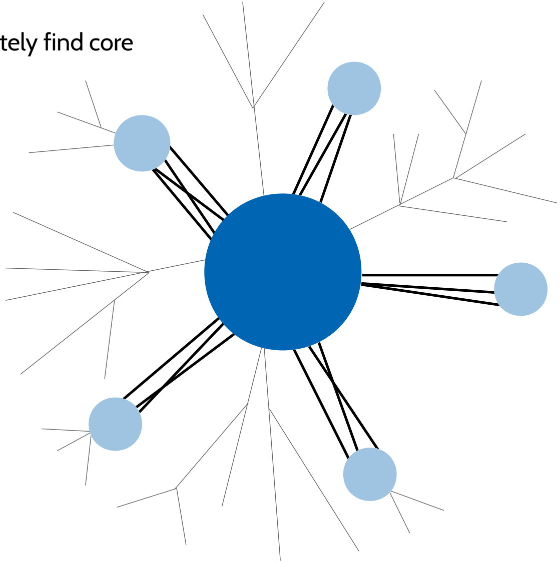


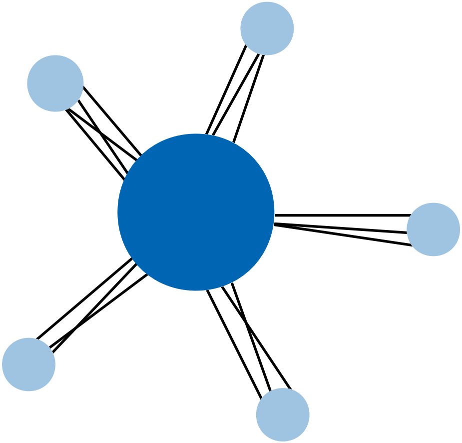
Algorithm

approximately find core



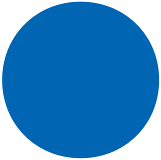
Algorithm

prune trees



Algorithm

prune protrusions

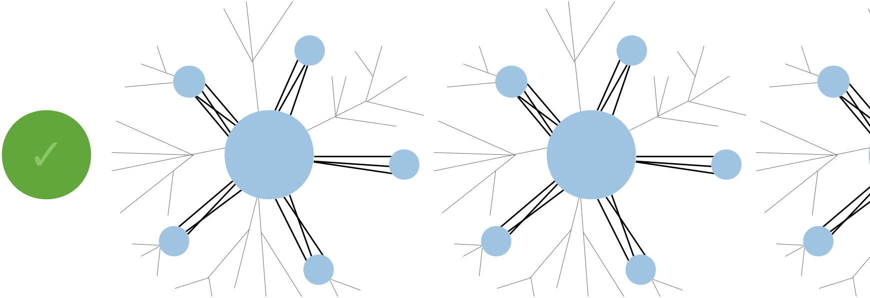


Algorithm

use brute force on core

Algorithm

repeat for every neighborhood

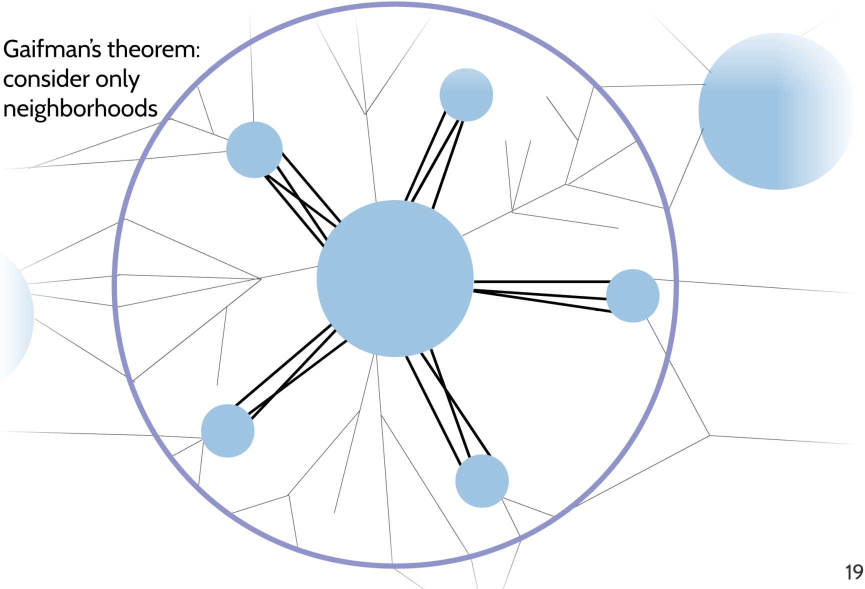


Algorithm

Done!

Practical Considerations I

Gaifman's theorem: consider only neighborhoods



Practical Considerations II

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- open question: can we do better in the average-case?

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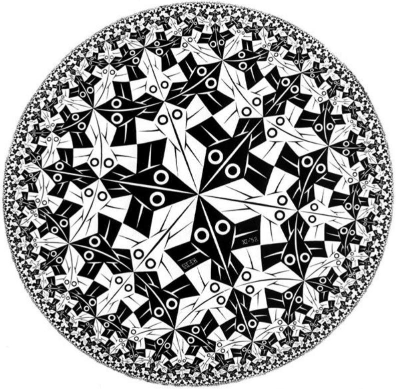
Big Question: model-checking on clustered models?

Want a Challenge?

Can we solve the problem on *Hyperbolic Random Graphs*?



[Cirle Limit I, M.C. Escher]

Lower Bounds

Average Case Complexity

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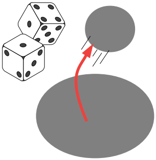
- worst-case: P and NP
- average-case: avgP and (NP, PComp)?
- expected time example:

$$\sum_{x \in \mathbf{N}} \mathrm{P}[X = x] \cdot \text{time}(x) = O(1)$$

With Colors



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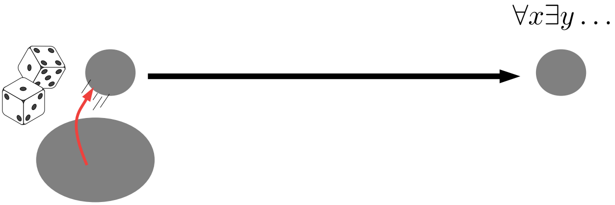

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$\forall x \exists y \ldots$

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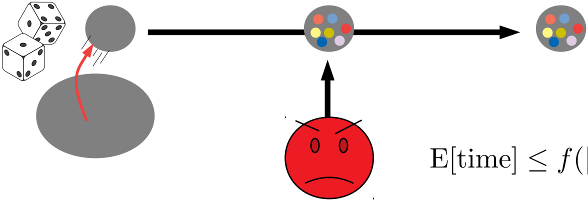

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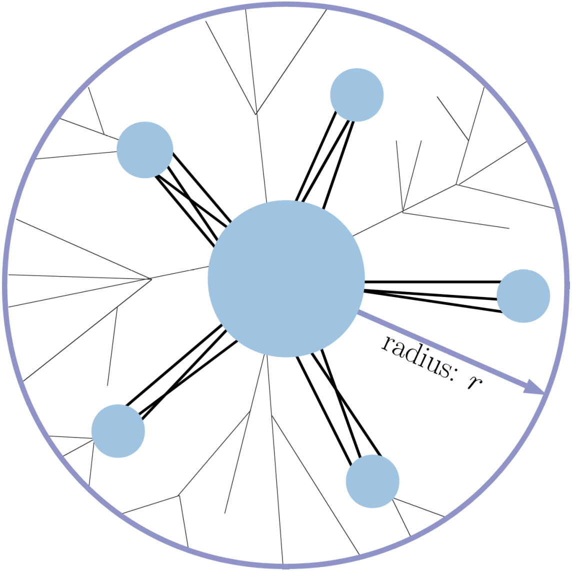
What if $\alpha < 3$?

[Dreier, Rossmanith 2019]



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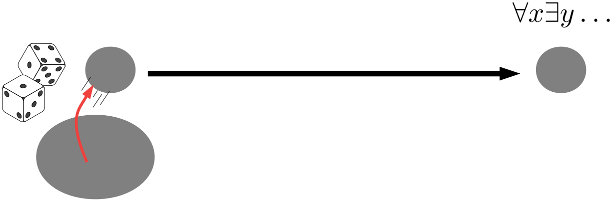

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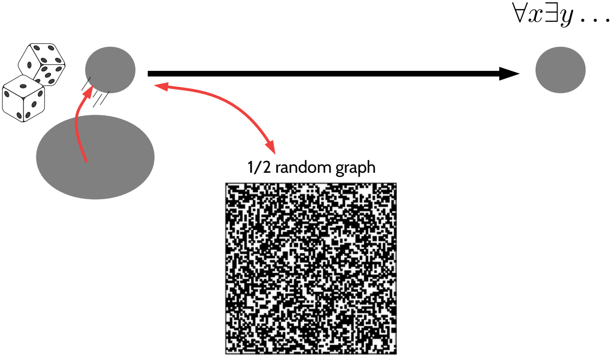

Zero-One Laws

1/2 random graph

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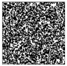
$\Pr[$  $\in$  $] \to 1$

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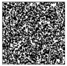
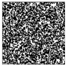
$\Pr[$  $\in$  $] \to 1$

$\Pr[$ apex vertex $\in$  $] \to 0$

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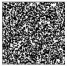
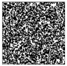
Pr[ $\in$ ] $\to 1$

Pr[apex vertex $\in$ ] $\to 0$

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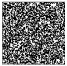
for all φ Pr[ $\models \varphi$] $\to \{0, 1\}$

Average-Case Hierarchy Collapse

- worst-case
  - W[1]-complete:

$$\exists x_1 \ldots \exists x_k$$

[Dreier, Lotze, Rossmanith 2020]

Average-Case Hierarchy Collapse

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Average-Case Hierarchy Collapse

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$$\exists x_1 \ldots \exists x_k \forall y \exists z$$

[Dreier, Lotze, Rossmanith 2020]

Average-Case Hierarchy Collapse

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- How about about (uncolored) lower bounds?

Thanks!

dreier@ac.tuwien.ac.at