Parameterized Algorithms WS 2021
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Date: December 20th, 2021

Exercise Sheet 08

The association of elves of Santa Claus is organizing a christmas party. Every elf has some other elves that he or she is friends with, but maybe they are not friends with everyone. The organizing comitee for the christmas party has k elves, and it is very important that every elf has at least one friend in the organizing comitee if they don't belong to the comitee themselves, this ensures that their christmas cookie preferences will be taken into account. Finding a good comitee is hard, even W[2]-hard, but this year Santa Claus had an idea. The elves will be divided into green elves and red elves, to fabricate toys for good and bad children respectively. Thus, as the elves must work together in the factory, they don't want to meet up with their colleagues after work, and they only maintain friendships with elves working on the other group. Santa belives that the choice of comitee will be made easier with this friendship division. What do you think?

Task T25
The Dominating Set problem is W[2]-complete in general but in many well-known graph classes it is fixed-parameter tractable. For instance, it has a linear kernel on the class of planar graphs (and, in fact, on graphs of bounded genus, on H-minor-free graphs etc.). A colleague claims that the problem is FPT on bipartite graphs. Would you agree with your colleague? Justify your answer.

Task T26
You are given an $n \times n$ matrix M and an integer parameter k. The goal is to select k non-zero entries S such that every other non-zero entry is either in the same row or same column as some element in S. Is this problem in FPT or W[1]-hard? Justify your answer.

Task T27
Prove that the Strong Exponential Time Hypothesis implies the Exponential Time Hypothesis.

Task H17 (5 credits)
The Induced Matching problem is to decide whether a given graph G has an induced matching of size at least k, where k is the parameter that is supplied as part of the input. While the Maximum Matching problem is polynomial-time solvable, the Induced Matching problem is NP-complete in general.
Is Induced Matching W[1]-hard or FPT on regular graphs? Prove it.

Task H18 (10 credits)
Show that assuming ETH implies that we cannot solve Dominating Set in time $f(k)n^{o(k)}$ for any computable function f (where k is the size of the dominating set and n the size of the graph).
Hint: The christmas log (Tió), in Catalunya, can help you with this problem. Just feed him clementines, and during christmas eve you can make it sh*t by singing the following song:

Caga Tió,
petit minyó
caga neules i torrons
i bones solucions
si no vols cagar et donaré un cop de bastó,
tió tió.

it will surely “bring” you some solutions for the problem.