RWTH Aachen **WS 2018**
Lehrgebiet Theoretische Informatik Problem Set 11
Dreier—Kuinke—Rossmanith 14.1.2018

Parameterized Algorithms Tutorial

Tutorial Exercise T1

Consider this problem:

- Input: A finite deterministic automaton M and a $k \in \mathbf{N}$
- Parameter: k
- Question: Is there a word of length at least k in $L(M)$?

Do not forget that the alphabet size of M can be arbitrarily large.

a) Is this problem NP-hard?

b) What is its parameterized complexity?

Tutorial Exercise T2

We are analyzing a variant of the *k-leaf spanning tree problem*. Instead of looking for a spanning tree with at least k leafs, we are looking for one with *exactly* k leafs.

a) Should we distinguish two versions of this problem: "exact k-leaf subtree" and "exact k-leaf spanning tree"? For the original problem the two versions where equally hard to solve.

b) What is the parameterized complexity of both problems?

c) Find an efficient algorithm if a variant is in FPT.

Tutorial Exercise T3

The MSO type of a structure S with a finite domain is the set of all MSO formulas ϕ with $S \models \phi$. Let us say that the q-type are the formulas in the type that have at most q variables. For simplicity we always assume that formulas are in prenex normal form.

a) Is the q-type of a structure finite or can it be infinite?

b) If it is infinite, are there only finitely many equivalence classes with regard to logical equivalence betweens formulas?

c) How could representatives of these equivalence classes look like?

Homework H1

Let t be a constant. Design an efficient algorithm that solves the following problem in polynomial time:

- Input: A graph G and a number k
- Output: A t-protrusion in G of size at least k or the answer that no such protrusion exists.

The degree of a polynomial that upper bounds the running time may depend on t.

Homework H2

Find a graph class that excludes some H as a topological minor, but contains *every* graph H as a minor (i.e., contains a graph that has H as a minor).