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Problem Set 7
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Parameterized Algorithms Tutorial

Tutorial Exercise T1

Show that Dominating Set $\leq_{\text{FPT}}$ Hitting Set.

Tutorial Exercise T2

Given a graph $G = (V, E)$, a *perfect code* for G is a vertex set $S \subseteq V(G)$ such that for all $v \in V(G)$ there is exactly one vertex in $N[v] \cap S$. The Perfect Code problem is defined as follows: given a graph $G = (V, E)$ and an integer parameter k, decide whether G has a perfect code with k vertices. This problem is W[1]-complete on general graphs. Show that this problem is fixed-parameter tractable if we assume that the input graph is planar. Use the fact that every planar graph has a vertex of degree at most five.

Tutorial Exercise T3

The r-Regular Vertex Deletion problem is defined as follows: given a graph G and an integer k, decide whether there is a set $S \subseteq V(G)$ of size at most k whose deletion results in an r-regular graph. A graph is r-regular if every vertex has degree exactly r. Show that this problem admits an algorithm with running time $O((r + 2)^k \cdot \text{poly}(n))$.

Homework H1

Show that Hitting Set $\leq_{\text{FPT}}$ Dominating Set.

Homework H2

The Partial Vertex Cover problem is defined as follows: given a graph G and integers k and t, decide whether there exists k vertices that cover at least t edges. The parameter is the integer t (when parameterized by k only, the problem is W[1]-complete). The point of this exercise is to use color-coding to obtain a randomized FPT-algorithm for this problem.

1. Show that if $t \leq k$ then the problem is polynomial-time solvable. What happens if the maximum degree of the input graph is at least t?
2. Now use the following idea for coloring the vertices of the graph with two colors, say, green and red. Assume that there exists $S \subseteq V(G)$ of size at most k such that S covers at least t edges. Color each vertex red or green with probability 1/2. Show that the probability that vertices in S are colored green and all vertices in $\{u \in V(G) \setminus S \mid (u, v) \in E(G) \text{ for some } v \in S\}$ are colored red is a function of k and t. Call such a coloring a *proper coloring*.

3. Given a properly colored graph, we now need to identify a solution quickly. Note that the green vertices decompose the graph into connected components and that these contain the potential solution vertices. Show that in a properly colored graph, the solution is always the union of some green components, that is, the solution either includes all vertices of a green component or none. Hence any green component with k or more vertices that does not cover at least t edges can be discarded. Use this to design an algorithm that identifies a solution set in a proper two-colored graph.

4. Use all the above facts to design a randomized FPT-algorithm for the problem and analyze its time complexity.