RWTH Aachen **WS 2018**
Lehrgebiet Theoretische Informatik Problem Set 3
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Parameterized Algorithms Tutorial

Tutorial Exercise T1

Given a graph $G = (V, E)$ and an integer k, you wish to add and/or delete a total of at most k edges such that the modified graph consists of just one clique (and a set of isolated vertices). Design an FPT-algorithm for the problem with parameter k.

Tutorial Exercise T2

Give a polynomial kernel for the following problem.

Input: A sequence of marbles, each with a non-negative integer weight and color.
Parameter: A positive integer k.
Question: Can we remove marbles of total weight at most k, such that for each color, all marbles of that color are consecutive?

Homework H1

The parameterized Dominating Set problem is this: Given a graph $G = (V, E)$ and an integer k, decide whether there exists a vertex subset $S \subseteq V$ of size at most k such that every vertex in $V \setminus S$ has a neighbor in S. Design an FPT-algorithm for this problem on graphs with maximum degree d, a constant.

Homework H2

You are given a boolean formula φ in CNF such that maximum clause size is q, and an integer k. Design an FPT-algorithm with parameter k that decides whether such a boolean formula has a satisfying assignment of weight at most k.

Homework H3

Show that the following problem is in FPT by the method of reducing to a problem kernel.

Input: A set $S = \{x_1, \ldots, x_n\}$ and a collection $\mathcal{C}$ of subsets of S.
Parameter: A positive integer k.
Question: Is there a collection $\mathcal{B}$ of subsets of S with $|\mathcal{B}| \leq k$ such that for each $A \in \mathcal{C}$, there exists a subcollection of $\mathcal{B}$ whose union is exactly A?

(**Hint.** In a yes-instance, each set A_i can be expressed a union of sets in $\mathcal{B}$. How many sets A_i can there be? Also if two distinct elements x_i and x_j occur in exactly the same sets of $\mathcal{C}$, how may we handle them?)