RWTH Aachen **WS 2018**
Lehrgebiet Theoretische Informatik Problem Set 1
Dreier—Kuinke—Rossmanith 22.10.2018

Parameterized Algorithms Tutorial

Tutorial Exercise T1

The Independent Set problem is defined as follows. Given a graph $G = (V, E)$ and an integer k, is ther a set S of size k such that for all $u, v \in S$ where $u \neq v$ it holds $uv \notin E(G)$? Is Independent Set restricted to graphs of maximal degree d, where d is a constant, fixed paramater tractable parameterized by the size of the solution k?

Solution

The problem is in FPT. One can design an FPT algorithm as follows. Take any node v. It will have degree d or lower. We know that either v or one of its neighbors must be in a maximal independent set. We branch over every possibility by taking a node of $N[v]$ into the independent set, deleting it and all of its neighbors and recursively solving the remaining graph. This gives us an algorithm with running time $O((d+1)^k \cdot n)$.

Tutorial Exercise T2

Since any planar graph is has a four coloring, any instance of the Planar Independent Set problem is guaranteed to have a solution of at least size $n/4$. Is the above guarantee version of the problem fixed parameter tractable paramertized by the solution size k?

Solution

If k is smaller than $n/4$ we can say yes. If $k > n/4$ it follows that $n < 4k$. This means that using any brute force algorithm on this instance will suffice.

Tutorial Exercise T3

The Cluster Vertex Deletion problem is defined as follows: given a graph $G = (V, E)$ and an integer parameter k, does there exist a set S of size at most k such that $G[V \setminus S]$ consists of a collection of disjoint cliques. The cliques are disjoint in the sense that they do not share vertices and/or edges and there is no edge with one endpoint in one clique and the other in a different clique. Design an algorithm that runs in FPT-time w.r.t. k as parameter.

Solution

A graph is precisely a cluster graph when it does not contain any induced paths of length three. Thus we can solve solve the problem in FPT time as follows. Either we are done, or we find an induced path of length three. One of these three nodes can not be in the solution. We branch over their deletion. This gives us an $O(3^k \cdot n^4)$ algorithm.

Homework H1

The TRIANGLE VERTEX DELETION problem is defined as follows. Given a graph $G = (V, E)$ and an integer parameter k, are there k vertices whose deletion results in a graph with no cycles of length three? Show that this problem is fixed-parameter tractable. What is the running time of your algorithm? Is there some easy way to improve the running time?

Solution

The idea now is to branch on the vertices of a triangle. Let (u, v, w, v) be a three cycle in G. We recurse on the instances $(G - u, k - 1)$, $(G - v, k - 1)$ and $(G - w, k - 1)$. The branching vector is $(1, 1, 1)$ and the running time is $3^k \cdot \text{poly}(|G|)$.

Homework H2

Implement the CLOSEST STRING algorithm that was presented in class and let it run on the example given. What is the run time? Ask a friend that does not know parameterized algorithms to design and implement an algorithm for the same problem and compare it with your implementation.