RWTH Aachen **SS 2014**
Lehrgebiet Theoretische Informatik Problem Set 11
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Parameterized Algorithms Tutorial

Tutorial Exercise T1

Irredundant Set is a W[1]-complete problem. The hardness is difficult to show. In this exercise, we show that it is in W[1]. An instance to this problem is a graph $G = (V, E)$ and an integer parameter k; the question is whether there is a set $V' \subseteq V$ of cardinality k having the property that each vertex $u \in V'$ has a private neighbor. The vertex u has a private neighbor x if $\{u, x\} \in E$ and for all $v \in V' \setminus \{u\}$, $\{v, x\} \notin E$.

Tutorial Exercise T2

The Induced Matching problem is to decide whether a given graph G has an induced matching of size at least k, where k is the parameter that is supplied as part of the input. While the Maximum Matching problem is polynomial-time solvable, the Induced Matching problem is NP-complete in general. Show by a reduction from Irredundant Set that this problem is also W[1]-hard in bipartite graphs.

Tutorial Exercise T3

We know that Clique is W[1]-complete on general graphs. This does not change when we restrict the graph class to be *regular*. Show that for any fixed integers $\alpha, c \geq 1$, the Clique problem with parameter k remains W[1]-hard on d-regular graphs with $d \geq \alpha k^c$. Conclude that Independent Set is W[1]-complete on regular graphs.

Homework H1

Consider the following problem: Given a graph $G = (V, E)$ and integers k and l, decide whether G has k vertices V' such that the cut $(V', V \setminus V')$ has at least l edges. The parameter is k. Show that this problem is W[1]-hard on d-regular graphs, where d is sufficiently large in comparison to k.

Homework H2

Show that Induced Matching remains W[1]-hard on regular graphs. **Hint:** Reduce from the k-Independent Set in regular graphs. Given an instance (G, k) of k-Independent Set where G is regular, construct a graph $\tilde{G}$ as follows: Take two copies of G, say G_1 and G_2, and for each vertex $v \in V(G)$, connect its copy $v_1 \in V(G_1)$ to the closed neighborhood $N[v_2]$ of its copy $v_2 \in V(G_2)$; and, vice versa: connect v_2 to all vertices in $N[v_1]$. Show that G has a k-independent set if and only if $\tilde{G}$ has a k-induced matching.