Parameterized Algorithms Tutorial

Tutorial Exercise T1

The problem 3-Colorability is defined as follows: Given a graph G decide if it is possible to assign every node of G one of three colors, such that no two nodes with the same color are adjacent. This problem is fpt parameterized by the treewidth of the graph. Give an algorithm that solves the problem given a tree decomposition of width w in time $O(3^w \cdot w \cdot n)$.

Tutorial Exercise T2

The problem Dominating Set is defined as follows: Given a graph G find the smallest set $S \subseteq V(G)$ such that every node of G is either in S or has a neighbor in S. This problem is fpt parameterized by the treewidth of the graph. Give an algorithm that solves the problem given a tree decomposition of width w in time $O(9^w \cdot w \cdot n)$.

Tutorial Exercise T3

The problem Hamiltonian Path is defined as follows: Given a graph G, does there exist a path that visits every node of the graph exactly once? Show there is an algorithm which solves the problem in linear time parameterized by treewidth, i.e. in time $f(w) \cdot n$ where w is the treewidth of G.

Homework H1

Let G be a graph and let $S \subseteq V(G)$ be some vertex subset. Show that the following properties are MSO_2-expressible:

- S is a vertex cover of G
- S is an independent set of G
- G is a connected graph
- S induces a cycle in G
- S induces an even cycle in G

Homework H2

The Irredundant Set problem is defined as follows: Given a graph G find a set $S \subseteq V(G)$ of maximal size, such that ever node in $u \in S$ has at least one neighbor $v \notin S$ such that $N(v) \cap S = \{u\}$. We call such a neighbor v a private neighbor. Show this problem is fpt parameterized by the treewidth of the graph without relying on Courcelle's Theorem.