

## Parameterized Algorithms Tutorial

### Tutorial Exercise T1

Consider the hereditary property  $\Pi_{0,0,k}$  which states that we can add less than  $k$  edges to  $G$  and make a chordal graph. Prove that  $\Pi_{0,0,k}$  has no finite forbidden set characterization.

### Tutorial Exercise T2

Consider the following: you are given  $n$  points in the plane and an integer  $k$ . Your task is to find  $k$  lines that cover all of the  $n$  points or conclude that no such set of lines exists. Design a quadratic kernel for this problem.

### Tutorial Exercise T3

Let us look at the UNIQUE HITTING SET problem which is defined as follows: Given is a universe set  $U$  and  $k$  sets  $X_1, \dots, X_k \subseteq U$ . Is there a set  $S \subseteq U$  such that for all  $1 \leq i \leq k$  it holds that  $|S \cap X_i| = 1$ ? Show by reducing to a kernel that this problem is in FPT when parameterized by the number of sets  $k$ .

### Homework H1

Show that planarity is a hereditary property. Is the forbidden set finite or infinite? If your answer is “finite” then construct the forbidden set; if your answer is “infinite”, then construct an infinite family  $\mathcal{F}$  of non-planar graphs such that

- for all  $G \in \mathcal{F}$ , all proper subgraphs of  $G$  are planar;
- for all distinct  $G_1, G_2 \in \mathcal{F}$ , we have that  $G_1$  is not an induced subgraph of  $G_2$ .

### Homework H2

We consider the CLUSTER EDITING problem: given a graph  $G$  and an integer  $k$ , decide whether one can add or delete up to  $k$  edges to turn  $G$  into a *cluster graph* (recall that a cluster graph is simply a collection of disjoint cliques).

Design a kernel with a quadratic number of vertices.

*Hint:* first prove that a graph is a cluster graph if and only if it does not contain an induced  $P_3$ , then design appropriate reduction rules.