RWTH Aachen | **WS 2011/12**
Lehrgebiet Theoretische Informatik | Problem Set 6
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Parameterized Algorithms Tutorial

Let G be a graph. A set of connected subgraphs $\mathcal{B}$ is called a *bramble* if the following condition holds:
For each $W_1, W_2 \in \mathcal{B}$ either W_1 and W_2 share a vertex *or* they are connected via an edge in G (i.e. there exist two vertices $u \in W_1, v \in W_2$ such that $uv \in E(G)$).
The *order* of $\mathcal{B}$ is the size of its minimum hitting set.

Tutorial Exercise T15

If a graph G has a bramble of order $k+1$, then $tw(G) \geq k$. Prove this by providing a winning strategy for a robber against k cops.
Show that a $r \times r$-grid has a bramble of order $r+1$.

Tutorial Exercise T16

Remember how we solved Vertex Cover on graphs of bounded treewidth? Do the same for Dominating Set. Then, think about how the algorithm could be modified in order to run in time $O(4^{tw(G)}\, poly(n))$.

Tutorial Exercise T17

Prove the following: if a graph G has treewidth k, there exists a vertex set $S \subseteq V(G)$ such that

1. $|S| \leq k$
2. The largest connected component in $G - S$ has size $\leq \frac{2}{3}n$

Homework H12

Show that graphs on the right have treewidth exactly k by

1. giving a strategy for $k+1$ cops to catch a robber or a tree decomposition of width k
2. providing a bramble of size $k+1$

If the strategy is simple enough you do not have to prove its correctness.

Graph	treewidth k
Tree	1
Cycle	2
Wheel	3
$K_{n,n}$	n

Homework H13

Proof that a graph G which has a vertex cover of size k also has treewidth at most k.

Homework H14

We define a n-loop as a $3n \times 3n$-grid where a $n \times n$-grid was removed from the center. Show that this graph has treewidth at most $2n$.