

## Parameterized Algorithms Tutorial

### Tutorial Exercise T13

Let  $G$  be a graph and  $S \subseteq V(G)$  some vertex subset. Show that the following properties are MSO-expressable:

- $S$  is a vertex cover of  $G$
- $S$  is an independent set of  $G$
- $G$  is a connected graph
- $S$  induces a cycle in  $G$
- $G$  has a hamiltonian path
- $S$  induces an even cycle in  $G$

### Tutorial Exercise T14

Which of the following graph properties are closed under taking minors?

- Acyclicity
- Chordality
- Planarity
- Bipartiteness
- Connectivity
- bounded degree
- having a  $\leq k$ -vertex cover

### Homework H10

Let  $G$  be a graph and  $S \subseteq V(G)$  some vertex subset. Show that the following properties are MSO-expressible:

- $S$  is a dominating set of  $G$
- $S$  induces a path in  $G$
- $S$  induces an even path in  $G$
- $S$  induces an odd cycle in  $G$
- $G$  is 3-colorable

### Homework H11

Which of the following graph properties are closed under taking minors?

- Bounded diameter
- Bounded average degree
- Distance  $k$  to planarity, i.e. one can delete at most  $k$  vertices from the graph to make it planar
- 3-Colorability
- excluding some fixed graph  $H$  as a minor

The average degree of a graph  $G = (V, E)$  is defined as  $d_{avg} = \frac{1}{|V|} \sum_{v \in V} d(v)$ . A classical result, the handshaking lemma, implies that it can also be calculated as  $d_{avg} = \frac{2|E|}{|V|}$