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Problem Set 3
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Parameterized Algorithms Tutorial

Tutorial Exercise T7

The 4-Coloring problem is the following: given a graph $G = (V, E)$, decide whether there is an assignment of colors to the vertices of G such that adjacent vertices receive distinct colors and the total number of colors used is at most four. This problem is known to be NP-complete.

Show that the property of 4-colorability is hereditary. Does this property admit a finite forbidden set? If not, construct an infinite family $\mathcal{F}$ of non-four-colorable graphs such that

- no two distinct graphs in $\mathcal{F}$ are induced subgraphs of one another;
- for all $G \in \mathcal{F}$, every proper subgraph of G admits a four-coloring.

Tutorial Exercise T8

Let k be a constant and consider the class of graphs that have a vertex cover of size *exactly* k. Does this class define a hereditary property? What can you say about the class of graphs that have a vertex cover of size *at most* k? In case you believe that the property is hereditary, how large is the forbidden set?

Homework H5

Show that planarity is a hereditary property. Is the forbidden set finite or infinite? If your answer is "finite" then construct the forbidden set; if your answer is "infinite", then construct an infinite family $\mathcal{F}$ of non-planar graphs such that

- for all $G \in \mathcal{F}$, all proper subgraphs of G are planar;
- for all distinct $G_1, G_2 \in \mathcal{F}$, we have that G_1 is not an induced subgraph of G_2.

Homework H6

A graph $G = (V, E)$ is a *divorce graph* if its vertex set can be partitioned into sets $X \uplus Y$ such that $G[X]$ is a complete graph and $G[Y]$ has no edges (there can be edges between the sets X and Y). Show that a graph is a divorce graph if and only if it does not contain the following two graphs as induced subgraphs: