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Tutorial 13
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Tutorial Exact Algorithms

Exercise T27 SINGLE MACHINE SCHEDULING. We are given a set of n tasks $T_1, \ldots, T_n$ and each task has associated with it three integers: a positive execution time $l(i)$; a non-negative release time $r(i)$; and, a positive deadline $d(i)$. We wish to determine whether there exists a single-machine schedule starting at time 0 such that the execution of each task takes place in the interval bounded by its release time and its deadline. A feasible solution may be viewed as a set $\mathscr{I}$ of disjoint open intervals on the real line such that the following hold:

1. Each interval I is assigned a label from the set $\{1, \ldots, n\}$ and if I is assigned label i, then $I = (a, a + l(i))$, where a is an integer satisfying $r(i) \leq a$ and $a + l(i) \leq d(i)$.
2. Each integer from $\{1, \ldots, n\}$ is a label of some $I \in \mathscr{I}$.

Use Inclusion-Exclusion to find out the number of feasible schedules.

Exercise T28 MAX INTERNAL SPANNING TREE. Let T be a tree with at least three vertices. A vertex $v \in V(T)$ is a leaf if $\deg(v) \leq 1$. A vertex is *internal* if it is not a leaf. If the tree is rooted at r, then the root is defined *neither* an internal vertex nor a leaf. The MAX INTERNAL SPANNING TREE problem is defined as follows:

Input: A connected graph $G = (V, E)$ and a positive integer $1 \leq c \leq |V|$.
Question: Is there a spanning tree of G with at least c internal vertices?

The objective of this exercise and Assignment H29 is to solve this problem using Inclusion-Exclusion.

1. We first establish a connection with branching walks. Show that: *There exists a spanning tree of G with at least c internal vertices iff there exists a vertex $s \in V$ and a branching walk $B = (T, \varphi)$ from s of length $|V| - 1$ such that $\varphi(V(T)) = V$ and T has at most $|V| - (c + 1)$ leaves.* Assume that $|V| \geq 3$.
2. We now count the number of branching walks with the properties in (1) above using Inclusion-Exclusion. To do this we need to define what the objects are and what properties they must satisfy. The objects in this setting are all branching walks $B = (T, \varphi)$ of length $n - 1$ such that T has at most $n - (c + 1)$ leaves. An object $B = (T, \varphi)$ has property A_v for $v \in V$ if $v \in \varphi(V(T))$. Formulate the problem of counting branching walks as an IE-expression.
3. For $X \subseteq V$, what does the term $\left|\bigcap_{u \in X} \bar{A}_i\right|$ represent? We will now evaluate this term. For $X \subseteq V$, $u \in V \setminus X$, $1 \leq l \leq n-1$ and $1 \leq q \leq n-(c+1)$, define $M_X(u, l, q)$ to be the set of all branching walks in $G[V \setminus X]$ from u of length l and with q leaves. Also define $m_X(u, l, q) := |M_X(u, l, q)|$. Write down $\left|\bigcap_{u \in X} \bar{A}_i\right|$ in terms of $M_X(u, l, q)$.
4. What is $m_X(u, 0, q)$? For $l \geq 1$, write down a recurrence for $m_X(u, l, q)$.

Homework Assignment H28 (10 Points) In the context of the Single Machine Scheduling problem, the total completion time of a schedule $T_{\pi(1)}, \ldots, T_{\pi(n)}$ is defined as time at which the last job $T_{\pi(n)}$ finishes. Use Inclusion-Exclusion to obtain the minimum completion time of a feasible schedule.

Homework Assignment H29 (10 Points) Complete the solution of the Max Internal Spanning Tree problem by proving that the recurrence for $m_X(u, l, q)$ is correct. Analyze the time and space complexity of evaluating $m_X(u, l, q)$ for a fixed $X \subseteq V$ and all u, l, q. What is the time and space complexity of the algorithm deciding Max Internal Spanning Tree?