

Tutorial Exact Algorithms

Exercise T19

Design an algorithm that solves MINIMUM EXACT SET COVER in time $O^*(2^{m/2})$ using Split&List.

Recall that this problem is defined as follows: given a finite set $\mathcal{U} = \{1, \dots, n\}$ and a collection \mathcal{F} of $m > 0$ subsets of \mathcal{U} , you have to find a subset $\mathcal{F}' \subseteq \mathcal{F}$ of minimum size that partitions \mathcal{U} .

Exercise T20

Show that VERTEX COVER cannot be solved in time $O(2^{o(n+m)})$ unless the ETH fails.

Exercise T21

Show that DOMINATING SET cannot be solved in time $O(2^{o(n+m)})$ unless the ETH fails.

Homework Assignment H19 (10 Points)

Let $a, b, c, b' \in \mathbb{N}^m$ and \leq_l the lexicographical ordering of vectors. Proof that $a + b > c$ and $b' > b$ implies $a + b' > c$.

Homework Assignment H20 (10 Points)

Proof that in Strassen's Algorithm, we have

$$\begin{aligned}C_{1,2} &= M_3 + M_5 \\C_{2,1} &= M_2 + M_4 \\C_{2,2} &= M_1 + M_3 + M_6 - M_2\end{aligned}$$

Homework Assignment H21 (10 Bonus Points)

Show that MAXIMUM LEAF SPANNING TREE cannot be solved in time $O(2^{o(n+m)})$ unless the ETH fails.

The MAXIMUM LEAF SPANNING TREE is defined as:

Input: A graph $G = (V, E)$, a number $k \in \mathbb{N}$

Question: Is there a spanning tree in G with at least k leaves.

Homework Assignment H22 (10 Bonus Points)

Design an algorithm that solves PARTIAL VERTEX COVER in time $O^*(2^{\omega n/3})$.

The PARTIAL VERTEX COVER is defined as:

Input: A graph $G = (V, E)$, numbers $k, t \in \mathbb{N}$

Question: Is there a $V' \subseteq V$ with $|V'| = k$ such that $|\{e \in E \mid e \cap V' \neq \emptyset\}| \geq t$