

## Tutorial Exact Algorithms

This tutorial is aimed primarily at practice in dynamic programming. The problems are all polynomial-time solvable.

**Exercise T15** MAXIMUM VALUE CONTIGUOUS SUBSEQUENCE. You are given a sequence  $a_1, \dots, a_n$  of  $n$  integers (positive and negative). You are required to find a *contiguous* subsequence for which the sum of the elements in the subsequence is maximized. Design a DP algorithm for this problem that does this in  $O(n)$  time. Note that in the absence of negative integers, the answer is trivially the whole sequence.

**Exercise T16** BOX STACKING. You are given  $n$  types of boxes, where the  $i$ th box has height  $h_i$  and a base of length  $l_i$  and width  $w_i$ . Assume without loss of generality that  $w_i \leq l_i$ . You want to create a stack of boxes that is as tall as possible, but you can stack box  $i$  above box  $j$  iff  $l_j > l_i$  and  $w_j > w_i$ . Design a DP algorithm for this problem.

**Homework Assignment H15 (10 Points)** LONGEST INCREASING SUBSEQUENCE. You are given a sequence of  $n$  integers  $a_1, \dots, a_n$  and are required to determine a subsequence (not necessarily contiguous) of maximum length such that the integers in the subsequence form a strictly increasing sequence. For example, if the input sequence is 14, 2, 1, 19, 4, 5, then an optimal subsequence is 1, 4, 5. Another optimal solution is 2, 4, 5.

**Homework Assignment H16 (10 Points)** BUILDING BRIDGES. Consider a 2-D map with a horizontal river passing through its center. There are  $n$  cities on the southern bank with  $x$ -coordinates  $a_1 < a_2 < \dots < a_n$  and  $n$  cities on the northern bank with  $x$ -coordinates  $b_1, b_2, \dots, b_n$ . You want to connect city  $i$  on the northern bank with city  $i$  on the southern bank by building bridges for as many north-south pairs of cities as possible with the restriction that no two bridges must cross. Design a DP algorithm to find out the maximum number of bridges that can be constructed.