RWTH Aachen University **WS 2010/11**
Lehrgebiet Theoretische Informatik Tutorial 4
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Tutorial Exact Algorithms

Exercise T6 (again)

The problem MaxCut is defined as follows:

Input:	A graph $G = (V, E)$.
Feasible solutions:	Any bipartition $V = V_1 \cup V_2$.
Goal:	Maximize the number of edges that are *cut* by the bipartition.

Design an algorithm or this problem with a running time of $O^*(\tau(3,3)^m)$, where as usual $m = |E|$.

Exercise T7

Let $F = \{C_1, \dots, C_m\}$ be a formula in 3-CNF with clauses $C_1, \dots, C_m$. Define for a variable x the "maxdegree" of x as $d(x) := \max\{\, |C| \mid x \in C\,\}$

Now consider the following simple branching algorithm for the 3SAT problem:

Given is a formula F in 3-CNF. If there is x that occurs only positively or negatively, or a clause $\{x\}$ or $\{\bar{x}\}$, choose the value for x accordingly. Otherwise choose x with maximal $d(x)$ and branch on the two possible assignments $x := 0$ and $x := 1$.

What is the running time in the form $O^*(c^n)$ when analyzed in the number n of variables?

Now consider a new measure. Let $\alpha_1, \alpha_2, \alpha_3 \in \mathbf{R}$ and define

$$\Phi(F) := \sum_{x \in vars(F)} \alpha_{d(x)}.$$

- Construct the branching vectors for $\Phi(F)$ for all possible cases.
- What are good choices for $\alpha_1, \alpha_2, \alpha_3$?
- What running time to we obtain?

Homework Assignment H7 (10 Points)

Consider a formula in F in 4-CNF and the same algorithm as in Exercise T6. Let again $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in \mathbf{R}$ and

$$\Phi(F) := \sum_{x \in vars(F)} \alpha_{\delta(x)}.$$

Compute, in terms of $\alpha_1, \alpha_2, \alpha_3$ and α_4, the branching vectors for all possible cases when measured in $\Phi(F)$.

Homework Assignment H8 (10 Points)

Find optimal values for $\alpha_1, \alpha_2, \alpha_3$ and α_4.