RWTH Aachen University | **WS 2010/11**
Lehrgebiet Theoretische Informatik | Tutorial 1
Kneis—Langer—Sikdar | 19.10.2010

Tutorial Exact Algorithms

Exercise T1

Let A be an algorithm with a running time of $f(n)$. Compute a bound on the input size of instances that can be solved by A in time t, $2t$, $100t$ and t^2.
Compute these values for $f(n) = n$, $f(n) = n^2$, $f(n) = 1.1^n$, and $f(n) = 2^n$.

Solution

t	$2t$	$100t$	t^2
$\sqrt{t}$	$\sqrt{2t}$	$\sqrt{100t}$	t
$\log_{1.1} t$	$\log_{1.1} t + \log_{1.1} 2$	$\log_{1.1} 100 + \log_{1.1} t$	$2 \log_{1.1} t$
$\log_2 t$	$\log_2 t + 1$	$\log_2 100 + \log_{1.1} t$	$2 \log_2 t$

Exercise T2

Compute the branching numbers for the following branching vectors:

$(3,3)$	$(2,2,2)$	$(6,6,3)$
$(1,2)$	(a,a)	$\underbrace{(a,a,\dots,a)}_{b \text{ times}}$

Solution

The root of $z^3 - 2$ is $2^{1/3}$. Similarly, the root of $z^2 - 3$ is $3^{1/2}$. Analogously, we obtain $2^{1/a}$ for (a,a) and $b^{1/a}$ for $\underbrace{(a,a,\dots,a)}_{b \text{ times}}$.
The characteristic polynomial for $(1,2)$ is $z^2 - z - 1$. The branching number is 1.619. $(6,6,3)$ is the same as $(3,3)$, as can be seen from the search tree.

Homework Assignment H1 (10 Points)

Design an algorithm that solves 3-Col in time $O^*(2^n)$.
3-Col is defined as follows:

Input:	Graph $G = (V,E)$
Question:	Is there are a partition $V_1 \cup V_2 \cup V_3 = V$ such that $G[V_i]$ does not contain any edge for $1 \le i \le 3$.

Solution

First we enumerate all possible $V_1 \subseteq V$. This takes $O^*(2^n)$. The graph is tree colorable, if for at least one V_1, the remaining graph is two-colorable. But this can easily be checked in polynomial time for any V_1.

Homework Assignment H2 (10 Points)

Compute the branching numbers of the following branching vectors (with an error of at most 1/100):

$(1,3)$	$(2,3)$	$(2,5)$
$(1,2,2)$	$(2,2,2,2)$	$(1,2,3,4,5)$

Preferably, you solve this exercise by writing a program that computes these numbers automatically.

Solution

≈ 1.466	≈ 1.325	≈ 1.237
2	2	≈ 1.966