RWTH Aachen | **WS 2010**
Lehrgebiet Theoretische Informatik | Exam
Kneis—Langer—Rossmanith—Sikdar | 23.3.2011

Exact Algorithms

Attempt any *four* questions. You may try the questions in any order.

Exercise 1 (10 Points) The E3-Set Splitting problem is, given a set of elements $U = \{1, \ldots, n\}$ and subsets $S_1, \ldots, S_m \subseteq U$ with $|S_i| = 3$, $1 \le i \le m$, to decide whether there is a bipartition U_1, U_2 of U, such that U_1, U_2 *splits* every set S_i, i.e., $S_i \cap U_1 \neq \emptyset$ and $S_i \cap U_2 \neq \emptyset$ for all S_i. Design an algorithm that solves E3-Set Splitting in time $O^*(c^n)$, where $c < 2$. Prove that your algorithm is correct and reaches the claimed running time.

Exercise 2 (10 points) Recall that if the Exponential Time Hypothesis (ETH) holds then one can show that the Vertex Cover problem on graphs with n vertices and m edges cannot be solved in time $O^*(2^{o(n+m)})$. Give a reduction from Vertex Cover to Dominating Set to prove that if ETH holds then Dominating Set cannot be solved in time $O^*(2^{o(n+m)})$ time on graphs with n vertices and m edges.

Exercise 3 (10 points) Given a graph $G = (V, E)$, the chromatic number of G is the smallest integer k such that V can be partitioned into a set of k independent sets. Design a dynamic programming algorithm that computes the chromatic number of an n-vertex graph in time $O^*(3^n)$. Next show how you can speed up your algorithm using the fact that an n-vertex graph has at most $3^{n/3}$ maximal independent sets and that there is an algorithm that enumerates these sets in time $O^*(3^{n/3})$.

Exercise 4 (10 points) You have k boxes each of integer volume B and n books with integer volumes $v(1), \ldots, v(n)$. You have to decide whether the books can be packed into the boxes given maintaining the size constraints. Use Inclusion-Exclusion to decide this problem in time $O^*(nB2^n)$. What is the space requirement of your algorithm? **Hint:** Define *objects* to be k-tuples $(\sigma_1, \ldots, \sigma_k)$, where each σ_i is a sequence from $\{1, \ldots, n\}$ such that $\sum_{a \in \sigma_i} v(a) \le B$.

Exercise 5 (10 points) Design an algorithm that solves Minimum Exact Set Cover in time $O^*(2^{m/2})$ using the Split & List technique. Recall that this problem is defined as follows: Given a finite set $\mathcal{U} = \{1, \ldots, n\}$ and a collection $\mathcal{F}$ of $m > 0$ subsets of $\mathcal{U}$, you have to find a subset $\mathcal{F}' \subseteq \mathcal{F}$ of minimum size that partitions $\mathcal{U}$.