RWTH Aachen | **WS 2025**
Theoretical Computer Science | Exercise 12
Rossmanith–Gehnen | 29.01.2026

Exercise for Analysis of Algorithms

Exercise 48

After convoluting a series $s_0, s_1, \ldots$ with itself, the result is a series with $b_n = 2^n$. How is s_n defined?

Exercise 49

Let

$$U(z) := \frac{1 - z - \sqrt{(1-3z)(1+z)}}{2z}.$$

Prove that $[z^n]U(z) = 3^n n^{O(1)}$ without doing any computations. Then find out what the constant in the monomial is, i.e., for what c is $[z^n]U(z) = \Theta(n^c 3^n)$.

Exercise 50

We continue with the last exercise where

$$U(z) = \frac{1 - z - \sqrt{(1-3z)(1+z)}}{2z}.$$

and we found the constant c with $[z^n]U(z) = \Theta(n^c 3^n)$.
Now also find the multiplicative constant in the Θ-notation, i.e., find a simple function $f(n)$ such that $[z^n]U(z) \sim f(n)$.

Exercise 51

In the lecture we used the saddle point method to approximate $[z^n]e^z$. In order to do it, we chose a circle as our integrating path.
Approximate now $[z^n]e^z$ using the same method but choosing a rectangular integrating path.
In order to simplify the calculation, you can use a degenerated rectangle.