RWTH Aachen **WS 2025**
Theoretical Computer Science Exercise 11
Rossmanith–Gehnen 22.01.2026

Exercise for Analysis of Algorithms

Exercise 43

In this exercise we consider the following (regular) CFG G:

$$S \to abA \mid bS \mid a$$
$$A \to bA \mid aS$$

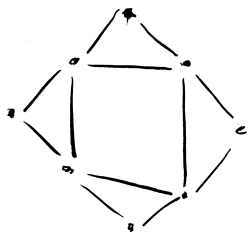
1. Find a generating function for number of words s_n in $L(G)$ that have length n.
2. What is the dominant singularity and what kind of singularity is it?
3. What is the exponential growth of s_n?
4. How precisely can you estimate s_n with just the knowledge of the dominating singularity and its nature?
5. Find a closed formula for s_n with an additive error of at most $O(0.8^n)$.

Exercise 44

An algorithm I computes an optimal independent set for an undirected graph $G = (V, E)$ of size n as follows: It picks a vertex v with maximal degree. If this degree is at most two, then the graph is a collection of cycles and paths and the solution is computed in linear time.

Otherwise, the optimal independent set either contains v (and then cannot contain any vertex in $N(v)$) or it does not. Hence, the algorithm recursively computes the two independent sets $I(G[V - N(v)])$ and $I(G[V - \{v\}])$ and then chooses the bigger one, or the first if they have the same size.

1. Simulate the algorithm on this graph:



2. Estimate its asymptotic running time up to a constant factor.

Exercise 45

Prove that

$$[z^n](1-z)^w \sim \frac{n^{-w-1}}{\Gamma(-w)}$$

for $w \in \mathbf{C}$ without using the theorem of the lecture. (The idea of this assignment is to get a deeper insight into the theorem.)

Hint: Use Newton's formula, then the first exercise on this sheet. Now replace the binomial coefficient by factorials or the gamma function. In the first case, you need to be careful with a definition of factorials for real numbers. In general, however, $\Gamma(n+1) = n!$.

Exercise 46

Approximate $[z^n]\frac{1}{2-e^z}$ up to an error of $O(12^{-n})$.

Exercise 47

Determine g_n up to an additive error of $O(4^n)$ for

$$G(z) = \sum_{n=0}^{\infty} g_n z^n = \frac{15z^2+8z+1}{15z^2-8z+1}.$$