**RWTH Aachen** | **WS 2025**
**Theoretical Computer Science** | Exercise 9
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## Exercise for Analysis of Algorithms

### Exercise 31

Compute:

$$\text{(a) } [z^n]\frac{1}{1+2z} \qquad \text{(b) } [z^n]\frac{z+1}{z-1} \qquad \text{(c) } [z^n]\left(\frac{z+1}{z-1}\right)^2 \qquad \text{(d) } [z^n]\frac{1}{\sqrt[3]{5+z}}$$

### Exercise 32

How many subsets of $\{1, \dots, 2000\}$ have a sum divisible by 5?
With generating functions at hand, you are able to solve this exercise. As this question seems to have a bit different nature than the questions we usually look at during our lecture, you have to think a bit outside of the box.
These could be guiding questions for you: For which sequence $(g_n)$ do you want to get a generating function? How can you get the answer for the exercise from the generating function? For this questions, maybe consider the (much easier) case where you want to know the number of subsets with a sum divisible by 2.

### Exercise 33

Find a function $f(n)$ in closed form such that

$$\prod_{k=1}^{n} k^k = f(n)(1 + O(1/n^2)).$$

Use Euler summation. It is okay if you cannot find the correct constant in the sum.

### Exercise 34

If you use Euler summation on a polynomial function, can you get an *exact* solution? Prove it or find a counterexample.

### Exercise 35

Approximate the following sum up to an error of $O(n^{-5})$:

$$\sum_{k=1}^{n} \frac{1}{k^2}$$

Find the constant $C$ in Euler's summation formula by looking up $\sum_{k=1}^{\infty} \frac{1}{k^2}$. Test your result for $n = 1000$. Use your favorite computing software.

### Exercise 36

Approximate the following sum up to an error of $O(n^{-5})$:

$$\sum_{k=1}^{n} \frac{1}{k^{5/2}}$$