RWTH Aachen | **WS 2025**
Theoretical Computer Science | 0. Exam
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Examination for Analysis of Algorithms

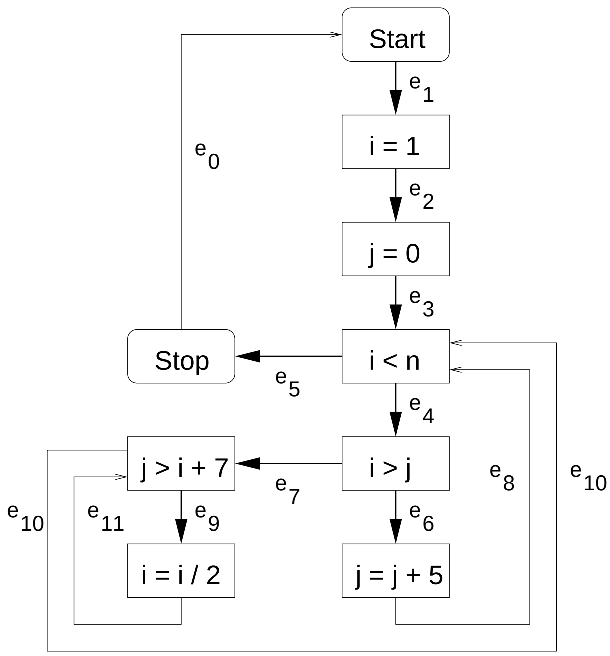
Exercise 1

```
i=1; j=0;
while (i<n)
if (i>j) j=j+5;
else while (j>i+7) i=i/2;
```

1. Draw the program flow graph,
2. mark a spanning tree and
3. identify the fundamental cycles!
4. How many non-trivial quantities must be determined for a complete analysis, and
5. which ones do you suggest for analysis?

Solution:

The spanning tree looks like this, with the thicker edges belonging to an example spanning tree:



From this, the following fundamental cycles can be derived:

$$\begin{aligned} C_0 &= e_0 + e_1 + e_2 + e_3 + e_5 && (1) \\ C_8 &= e_8 + e_4 + e_6 && (2) \\ C_{10} &= e_{10} + e_4 + e_7 && (3) \\ C_{11} &= e_{11} + e_9 && (4) \end{aligned}$$

Since e_0 is always 1, we only need to determine three non-trivial quantities. Obviously, the equations $E_0 = E_1 = E_2 = E_3 = E_5 = 1$, $E_6 = E_8$, $E_7 = E_{10}$, and $E_9 = E_{11}$ hold. Therefore, it suffices to determine, for example, the three independent quantities E_6, E_7, and E_9.

Exercise 2

For an arbitrary algorithm that searches a sorted array using comparisons, let C^+ denote the number of comparisons for a successful search and C^- for an unsuccessful search (elements and gaps occur with equal probability).

How do C^+ and C^- asymptotically relate to each other?

Solution:

In this context, we easily obtain the equation

$$C^- = (C^+ + 1)\left(1 - \frac{1}{|T| - 1}\right).$$

From this, it follows that

$$C^- = (C^+ + 1)\left(1 + O\left(\frac{1}{|T|}\right)\right).$$

Exercise 3

In a monastery, monks can either pray or work under the following conditions: Each hour, they can either pray or work, but they cannot work for more than two consecutive hours. Each hour of prayer earns two *Bräuros* and each hour of work earns one *Bräuro*.

- Determine the OGF $M(z)$ for the number of ways to earn exactly n Bräuros.
- What is the exponential growth (i.e., for which a does $M_n \asymp a^n$)?

Solution:

The OGF is given by

$$M(z) = \frac{1 + z + z^2}{1 - z^2 - z^3 - z^4}.$$

The exponential growth can be determined as

$$M_n \asymp a^n$$

for

$$a \approx 1.4656.$$

Exercise 4

Find an asymptotic estimate for f_n, where

$$f(z) = \frac{e^z\sqrt{1+2z}}{\sqrt{1-3z}}$$

is the generating function for f_n.

Solution:

Using Darboux's theorem, we obtain

$$[z^n]f(z) \sim \frac{e^{1/3}(1+2/3)^{1/2}}{\Gamma(1/2)(1/3)^n} = \frac{(5/3)^{1/2}e^{1/3}}{\Gamma(1/2)}\frac{3^n}{\sqrt{n}}.$$