Analysis of Algorithms WS 2022
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Old Exam (2020) 0

This is an old exam from 2020.

Task K1 (10 Points)
Order the following four power series by their asymptotic growths. Justify your answer.

a) $[z^n]e^{z+z^2}$

b) $[z^n]e^{z+z^2/2}$

c) $[z^n]\sqrt{1-z-z^2}$

d) $[z^n]1/\sqrt{1-z-z^2}$

Task K2 (10 Points)
Solve the following recurrence relation:

$$a_n = n + 1 + \frac{1}{n}\sum_{k=0}^{n-1} a_k \text{ for } n > 0 \text{ and } a_0 = 2$$

Task K3 ($1 + 7 + 2$ Points)
Consider the following context-free grammar G:

$$S \to aSbS \mid cSdS \mid \epsilon$$

a) Write down all words up to length four of $L(G)$.

b) Find out whether the number of words of length up to n grows asymptotically faster or slower than 3^n. Justify your answer.

c) The generating function has two dominant singularities on the real axis. Explain why this is normally not the case but happens here.

Task K4 (10 Points)
Consider the problem *Triangle Deletion:*

Input: A graph G and budget $k \in \mathbf{N}$.
Output: Yes iff there is a set $W \subseteq V(G)$ and a set $F \subseteq E(G)$ such that $2|W| + |F| \leq k$ and $G - W - F$ is triangle-free.

We propose the following branching algorithm $\mathcal{A}(G, k)$ for this problem.

1. If $k < 0$, return NO.
2. If G is triangle-free, return YES.
3. Otherwise, find a triangle $\{v_0, v_1, v_2\}$ in G.
4. Call $\mathcal{A}(G - v_i, k - 2)$ for each $0 \leq i \leq 2$.
5. Call $\mathcal{A}(G - e, k - 1)$ for each edge $e \in \{\{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_1\}\}$.
6. If any of the recursive calls returns YES, return YES. Otherwise NO.

Analyze the number of recursive calls in the worst-case for a given budget k. The exponential growth of the number of recursive calls is precise enough.