Analysis of Algorithms WS 2022
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Exercise Sheet 12

Tutorial Exercise T12.1

Let

$$U(z) := \frac{1 - z - \sqrt{(1-3z)(1+z)}}{2z}.$$

Prove that $[z^n]U(z) = 3^n n^{O(1)}$ without doing any computations. Then find out what the constant in the monomial is, i.e., for what c is $[z^n]U(z) = \Theta(n^c 3^n)$.

Tutorial Exercise T12.2

In the lecture we used the saddle point method to approximate $[z^n]e^z$. In order to do it, we chose a circle as our integrating path.
Approximate now $[z^n]e^z$ using the same method but choosing a rectangular integrating path.
In order to simplify the calculation, you can use a degenerated rectangle.

Homework Exercise H12.1

We continue exercise T12.1 where

$$U(z) = \frac{1 - z - \sqrt{(1-3z)(1+z)}}{2z}.$$

and we found the constant c with $[z^n]U(z) = \Theta(n^c 3^n)$.
Now also find the multiplicative constant in the Θ-notation, i.e., find a simple function $f(n)$ such that $[z^n]U(z) \sim f(n)$.

The following exercise is from a former exam:

Homework Exercise H12.2

Consider the following context-free grammar G:

$$S \to aSbS \mid cSdS \mid \epsilon$$

a) Write down all words up to length four of $L(G)$.

b) Find out whether the number of words of length up to n grows asymptotically faster or slower than 3^n. Justify your answer.

c) The generating function has two dominant singularities on the real axis. Explain why this is normally not the case but happens here.