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Exercise Sheet 11

Tutorial Exercise T11.1

Let $z \in \mathbf{C}$. How can we write $z^n + \bar{z}^n$ using only real numbers if $z = Re^{i\phi}$?

Tutorial Exercise T11.2

In this exercise we consider the following (regular) CFG G:

$$S \to abA \mid bS \mid a$$
$$A \to bA \mid aS$$

1. Find a generating function for the number of words s_n in $L(G)$ that have length n.

2. What is the dominant singularity and what kind of singularity is it?

3. What is the exponential growth of s_n?

4. How precisely can you estimate s_n with just the knowledge of the dominating singularity and its nature?

5. Find a closed formula for s_n with an additive error of at most $O(0.8^n)$.

Homework Exercise H11.1

Prove that

$$[z^n](1-z)^w \sim \frac{n^{-w-1}}{\Gamma(-w)}$$

for $w \in \mathbf{C}$ without using the theorem of the lecture. (The idea of this assignment is to get a deeper insight into the theorem.)

Hint: Use Newton's formula and replace one of the implicit factorials by a gamma function. Remember that $\Gamma(n+1) = n!$.

Homework Exercise H11.2

Approximate $[z^n]\frac{1}{2-e^z}$ up to an error of $O(12^{-n})$.