

Exercise Sheet 08

Due date: next tutorial session, preferably in groups

Tutorial Exercise T8.1

Here is a *classical* problem: n gentlemen attend a Christmas party and check their hats. As it can happen with *gentlemen* at Christmas parties, they have a little too much drink and the checker returns the hats at random. What is the probability that no gentlemen receives his own hat? How does the probability depend on the number of gentlemen?

Tutorial Exercise T8.2

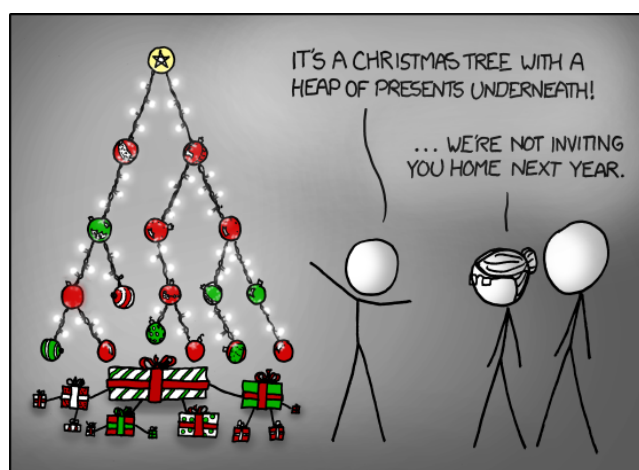
Find a bivariate generating function and a closed-form expression for the number of bitstrings of length n that contain exactly m ones and do not contain the substring 11.

Homework Exercise H8.1

Bots became quite creative these days. During the Christmas season, they will spam users with Christmas greetings of the following form:

$$P \rightarrow \text{👶} P \text{❄️} \quad | \quad \text{🎄} P \text{🕯️} \quad | \quad \text{❤️} \quad | \quad \text{❤️} P$$

How many unique messages of length n can you get at most? Particularly interesting values are $n = 4096$, as these are the maximal lengths of text messages on WhatsApp and Telegram. Use the symbolic method. Emojis are the terminal symbols and capital letters are variables.



xkcd 835: *Not only is that terrible in general, but you just KNOW Billy's going to open the root present first, and then everyone will have to wait while the heap is rebuilt.*

Homework Exercise H8.2

We will again look at the question from H3.2:

How many subsets of $\{1, \dots, 2000\}$ have a sum divisible by 5?

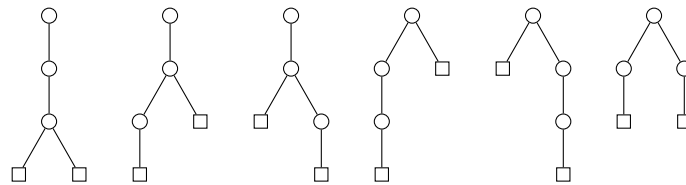
With generating functions at hand, you are able to solve this exercise. As this question seems to have a bit different nature than the questions we usually look at during our lecture, you have to think a bit outside of the box.

These could be guiding questions for you: For which sequence (g_n) do you want to get a generating function? How can you get the answer for the exercise from the generating function? For this questions, maybe consider the (much easier) case where you want to know the number of subsets with a sum divisible by 2.

Homework Exercise H8.3

Find a bivariate generating function for the number of rooted, oriented trees with exactly n internal and m external vertices $T_{n,m}$. For what values of n, m do we have $T_{n,m} = T_{m,n}$?

Example: $b_{3,2} = 6$ and these are the six trees with 3 internal and 2 external nodes:



As a warmup exercise try to find and draw all trees with 2 internal and 3 external nodes.

Hint: Do not do all the computations by hand. Seek the help of a computer algebra system. `maxima` or `WolframAlpha` can solve quadratic equations and can find the coefficients of a generating function via Taylor expansion.