Analysis of Algorithms WS 2022
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Exercise Sheet 06

Due date: next tutorial session, preferably in groups

Tutorial Exercise T6.1

Consider the following algorithm that searches an element x in a sorted array a of length $n = km + 1$:

```
i:= 1;
while a[i]<=x
  if a[i]=x then return i;
  i:=i+m;
  if i>n return 0;
for j=i-1 downto max(1,i-(m-1))
  if a[j]=x then return j;
if a[j]<x then return 0;
return 0;
```

a) Draw the search tree and compute the internal and external path length for $n = 10$ and $m = 3$.

b) Determine C^+ and C^- for arbitrary m, k.

c) What is, for given n, the best choice for m w.r.t. the running time?

Homework Exercise H6.1

We continue to look at the binary words defined in H5.3. Élisabeth Philippe Marie Hélène de Bourbon wants to write a program that generates such words. Let W_n be the set of all well-formed words of length n. The program should output one of the words randomly—such that every word in W_n is output with the same probability. Daniel's method from H5.3 turned out to be too slow for large n.

Invent a method to generate such a word in time $O(n^2)$ and implement it. Do not forget that just adding two n-bit numbers takes time $\Theta(n)$.

Homework Exercise H6.2

Use summation factors to solve the following recurrence:

$$\begin{aligned} a_0 &= 0 \\ a_n &= \frac{a_{n-1}}{n} + \frac{1}{(n-1)!} \qquad \text{for } n \geq 1 \end{aligned}$$