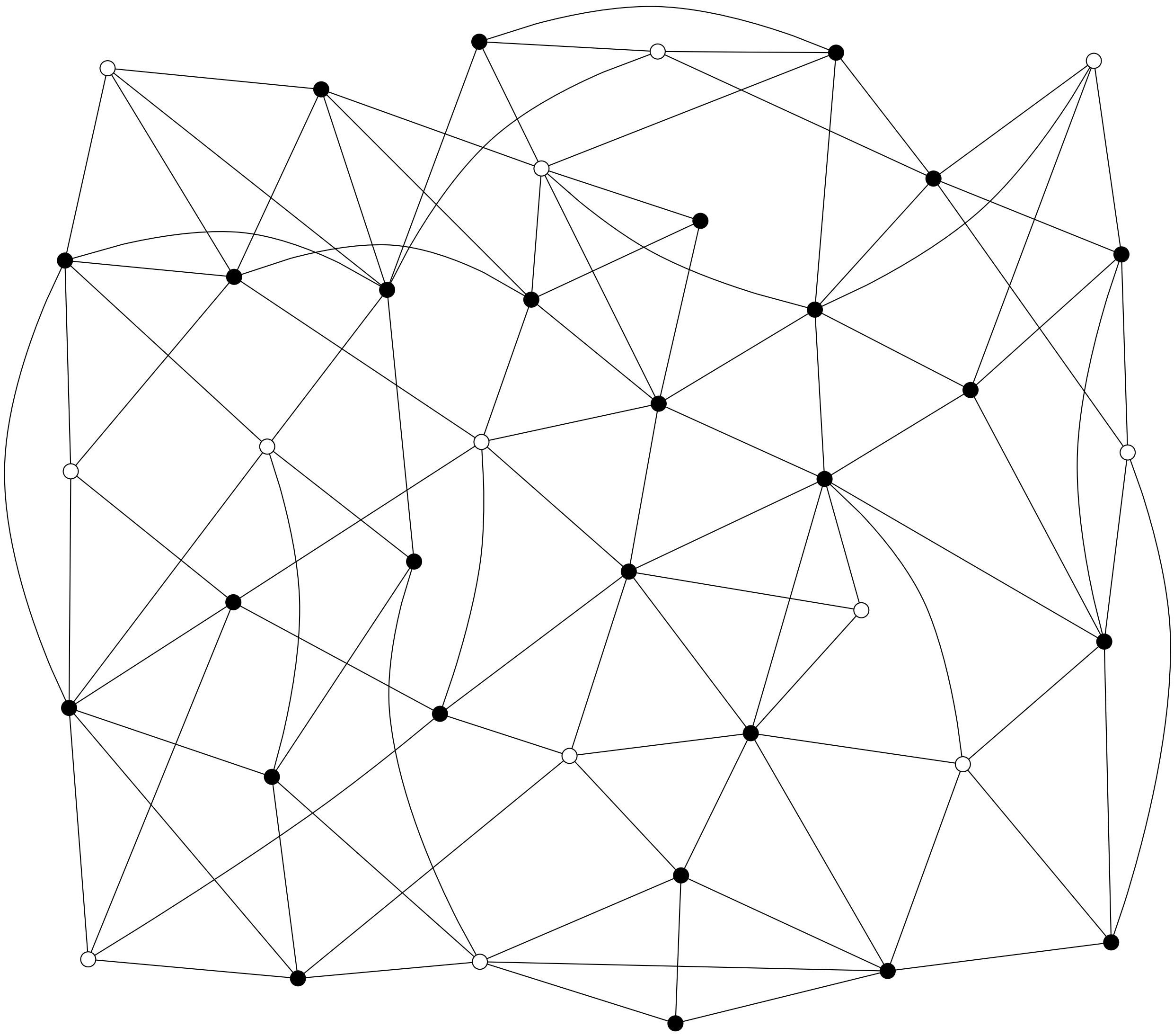
Date: November 21, 2022

# Exercise Sheet 04

Due date: next tutorial session

### Tutorial Exercise T4.1

A *vertex cover* of an undirected graph $G = (V, E)$ is a subset $C \subseteq V$ of its vertices such that at least one endpoint of every edge is in $C$, i.e., for every $(v_i, v_j) \in E$, either $v_i \in C$ or $v_j \in C$. Informally speaking, "$C$ covers all edges." It is an NP-complete problem to find out whether a graph has a vertex cover of a given size. The following example shows a graph of order 40. The black vertices comprise a minimum size vertex cover.



Now, given a graph $G = (V, E)$ we define its size as the number of vertices $|V|$ of the graph. So, let us consider a graph of size $n$ and let $k$ be targeted vertex cover size. Find an algorithm for Vertex Cover that runs in time $1.5^k n^{O(1)}$. Hints: If a graph has maximum degree two, i.e., without any vertex of degree three or more, this problem can be solved in polynomial time. On the other hand, if a vertex $v$ is not in the vertex cover, all of its neighbors have to be there.

**Tutorial Exercise T4.2**

Solve the following recurrence: Let $a_0 = 1$, $a_1 = 1$, $a_2 = 4$ and

$$a_n = 2a_{n-1} - a_{n-2} + 2a_{n-3}, \text{ for } n \geq 3.$$

**Tutorial Exercise T4.3**

Given an array $a$ of length $n$, an algorithm compares all pairs $(a[i], a[j])$ for all $i < j \leq n$, and then calls itself recursively on all proper prefixes of $a$.

How often does the algorithm compare two pairs? Use the repertoire method!

**Homework Exercise H4.1**

Solve the following recurrence: Let $a_0 = 0$, $a_1 = 3$ and

$$a_n = 4a_{n-1} - 4a_{n-2} \text{ for } n > 1.$$

**Homework Exercise H4.2**

Use the repertoire method to find a closed form for the following recurrence:

$$\begin{aligned} a_0 &= 5 \\ a_1 &= 9 \\ a_n &= na_{n-1} + n^2 a_{n-2} - n^4 - 3n^2 + 5 \qquad \text{for } n \geq 2 \end{aligned}$$

**Homework Exercise H4.3**

Solve the following recurrence and find a nice representation of the solution (in a mathematical sense).

$$\begin{aligned} c_0 &= 2 \\ c_1 &= 4 \\ c_n &= c_{n-2}^{\log c_{n-1}} \end{aligned}$$

Hint: Let $F_n$ be the $n$th Fibonacci number. Write $c_n$ as some function of $F_n$.