Analysis of Algorithms WS 2022
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Exercise Sheet 03

Due date: next tutorial session

Tutorial Exercise T3.1

If a flow diagram consists of n nodes and m edges, how many fundamental cycles do we get?

Tutorial Exercise T3.2

Prove or disprove: In every flow diagram you can find a spanning tree such that all fundamental cycles contain only edges that are labeled with plus.

Tutorial Exercise T3.3

In this exercise, we consider Prim's Algorithm, which computes a minimum spanning tree. The input to this algorithm is a graph $G = (V, E)$, a weight function on the edges $w\colon E \to \mathbf{R}$ and a starting node r.

```
 1  for each u ∈ V do
 2      key[u] ← ∞
 3      π[u] ← NIL
 4  key[r] ← 0
 5  M ← V
 6  while (M ≠ ∅) do
 7      u ← min-from(M)
 8      for each v ∈ neighbors(u) do
 9          if (v ∈ M) ∧ (w(u, v) < key[v]) then
10              π[v] ← u
11              key[v] ← w(u, v)
```

Construct the control flow graph, a spanning tree in the control flow graph, the fundamental cycles, a corresponding linear system of equations and a solution to this system.

Homework Exercise H3.1

Consider the following program:

```
int sel_sort ( int a[], int n ) {
  for ( int i = 0; i < n; ++i ) {
    int min = i;
    for ( int j = i; j < n; ++j ) {
      if ( a[j] < a[min] ) {
        min = j;
      }
    }
    int temp = a[i];
    a[i] = a[min];
    a[min] = temp;
  }
}
```

The input to this program is an array $a[0, \ldots, n-1]$ that contains n pairwise distinct integer keys in random order.

a) Explain how this program sorts the given array.

b) Analyse how often each instruction of the program is executed on average depending on n.

c) There is only one instruction whose analysis is not trivial. Which one is it?

   Make a table for small values of n by hand that lists the results for this instruction. Compare the table entries with the results from your closed formula that you obtained in b).

Homework Exercise H3.2

Try to solve the following puzzle: How many subsets of $\{1, \ldots, 2000\}$ have a sum divisible by 5?