Analysis of Algorithms WS 2022
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Exercise Sheet 01

Due date: 6 November 14:30

Tutorial Exercise T1.1

Consider the following algorithm that computes the maximum element in an array of positive integers. We assume that all elements are pairwise different and that each permutation occurs with equal probability.

```
int maxElem(int *a, int N)
{
  int i,max;

  max = -1;              /*  1  */
  for (i=0; i<N; i++)    /*  2  */
    if (a[i] > max)      /*  3  */
      max = a[i];        /*  4  */
  return max;            /*  5  */
}
```

- How often are the lines 3 and 4 executed in the *worst case* (in the *best case*)?

- What is the probability that this worst case occurs?

- How often are the lines 3 and 4 executed in the *average case*?

Tutorial Exercise T1.2

Let w be a random word in $\{a, b\}^n$ chosen independently and with uniform probability. What is the expected number of iterations of the while-loop in the following algorithm? The function is_palindrome checks if the given word is a palindrome, i.e., if the word and its reverse are identical.

```
i = 2;
while (i <= n)
   if (is_palindrome(w[1],...,w[i]))
       return true;
   i++;
return false;
```
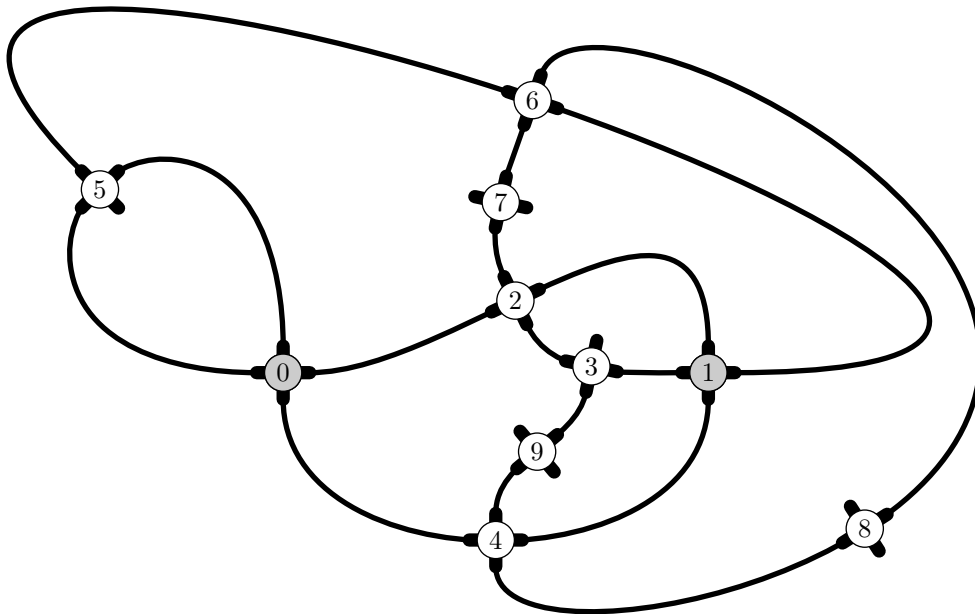
Homework Exercise H1.1

RWTH Aachen University has an exclusive contract with the well-known *Uranus Corporation* on the delivery of canal and pump supplies. Unfortunately, the responsible person failed to notice that Uranus sells only the "one-fits-all" product KKuRPSE (Kanalkopplungsundregulie-rungspumpstationeinheit). Such a KKuRPSE is a cylinder with four connectors placed north, east, south, and west of the cylinder (see the figure).

Now, the Institute for Tunnel and Canal Construction (ITCC) is conducting some research: The researchers first place n KKuRPSEs on a big green. Then they start to connect KKuRPSEs as follows: They digg a new canal between two arbitrary connectors. The new canal cannot cross another existing canal, of course. They then place a new KKuRPSE somewhere into this new canal, such that exactly two connectors on opposite sides of the new KKuRPSE are now attached to the canal.

The question is: How often can this connection procedure be repeated depending on the number n of initial KKuRPSEs on the green?

Example

Assume the researchers start with two KKuRPSEs, here labeled 0 and 1. Then a couple of connection steps are executed, where each new KKuRPSE receives consecutive numbers until no more connections are possible.

Homework Exercise H1.2

Two natural numbers $m \neq n$ are called *friendly*, if the sum of all factors of m equals n — or the other way round. A son and a father wrote the following programs that compute friendly numbers. What are their running times? Assume that the constant is replaced by N.

Son

```cpp
#include <iostream>
int e[150000];
int realdiv(int a) {
  int n=0;
  for(int i=1; i+i<=a; i++)
    if(a%i==0) n+=i;
  e[a] = n;
  return n;
}
main() {
  for(int i=0; i<150000; i++) {
    int a = realdiv(i);
    if(a >= i) continue;
    if(e[a]==i) {
      std::cout << i << " "
        << realdiv(i) << "\n";
    }
  }
}
```

Father

```c
#include <stdio.h>
#define N 1000000
int factorsum[N];
int main() {
  int i;
  for(i=1; i<N; i++) {
    int p=i;
    while(p<N) {
      factorsum[p] += i;
      p += i;
    }
  }
  for(i=1; i<N; i++) {
    int a = factorsum[i]-i;
    if(a<i && i==factorsum[a]-a)
      printf("%d %d\n", a, i);
  }
  return 0;
}
```