Analysis of Algorithms, WS 2020
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Date: 10 February 2021

Exercise Sheet 13

This is an old exam from 2014.

Problem T29

Consider the following algorithm for searching an array $a[1, \ldots, n]$ for an element x. We assume that the array is sorted in increasing order and that the element x is at some random location in the array. Let B_n be the expected number of comparisons on an n-element array. Write down a recurrence for B_n. What is B_3?

Algorithm: Binary Search with randomly chosen pivot element

1. Choose randomly and with uniform probability an $i \in \{1, \ldots, n\}$.
2. If $a[i] = x$, output i and halt.
3. Continue recursively on the left subarray, if $x < a[i]$, or the right subarray, if $x > a[i]$.

Problem T30

An alphabet Σ consists of two numeric characters $1, 2$ and four alphabetic characters a, b, c, d. Find and solve a recurrence relation for the number of words of length n in Σ^*, where there are no consecutive (identical or distinct) numeric characters.

Problem T31

Find an expression for

$$[z^n] \frac{1}{(1-z)^2} \ln \frac{1}{1-z}.$$

Your solution can include a sum!

Problem T32

Sort the series with the following generating functions by their asymptotic growth. Justify your steps!

1. $A(z) = \frac{1}{\sqrt{2-\frac{1}{z}}}$.
2. $B(z) = \frac{z}{2-3z+z^2}$.
3. $C(z) = \frac{e^{-z-z^2/2}}{1-z}$.