Analysis of Algorithms, WS 2020
Prof. Dr. P. Rossmanith
Dr. E. Burjons, H. Lotze, D. Mock

RWTH AACHEN UNIVERSITY

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Exercise Sheet 12

Problem T28

In the lecture we used the saddle point method to approximate $[z^n]e^z$. In order to do it, we chose a circle as our integrating path.
Approximate now $[z^n]e^z$ using the same method but choosing a rectangular integrating path.
In order to simplify the calculation, you can use a degenerated rectangle.

Problem H28 (10 credits)

In this exercise we consider the following (regular) CFG G:

$$S \to abA \mid bS \mid a$$
$$A \to bA \mid aS$$

1. Find a generating function for number of words s_n in $L(G)$ that have length n.
2. What is the dominant singularity and what kind of singularity is it?
3. What is the exponential growth of s_n?
4. How precisely can you estimate s_n with just the knowledge of the dominating singularity and its nature?
5. Find a closed formula for s_n with an additive error of at most $O(0.8^n)$.

Problem H29 (10 credits)

In the lecture, we used the exponential generating function $I(z)$ for the number of involutions to demonstrate the power of the saddle point method. In this exercise, you should derive this EGF. Remember, an *involution* is a permutation which is self-inverse.
First, find the recurrence relation for I_n where I_n is the number of involutions over n elements. Then use the usual toolbox for EGF to find an ordinary differential equation for $I(z)$. You can solve this by using tools like Wolfram Alpha or use that $I(z) = ce^{f(z)}$.