Analysis of Algorithms, WS 2020
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Exercise Sheet 04

Problem T9

Given an array a of length n, an algorithm compares all pairs $(a[i], a[j])$ for all $i < j \leq n$, and then calls itself recursively on all proper prefixes of a.

How often does the algorithm compare two pairs? Use the repertoire method!

Problem T10

Solve the following recurrence relation by order reduction:

$$a_0 = 8000 \qquad a_1 = \frac{1}{2} \qquad a_{n+2} + a_{n+1} - n^2 a_n = n!$$

Problem H8 (10 credits)

Use the repertoire method to find a closed form for the following recurrence:

$$\begin{aligned} a_0 &= 5 \\ a_1 &= 9 \\ a_n &= n a_{n-1} + n^2 a_{n-2} - n^4 - 3n^2 + 5 \qquad \text{for } n \geq 2 \end{aligned}$$

Problem H9 (20 credits)

We continue to look at the binary words defined in H7. Élisabeth Philippe Marie Hélène de Bourbon wants to write a program that generates such words. Let W_n be the set of all well-formed words of length n. The program should output one of the words randomly—such that every word in W_n is output with the same probability. Daniel's method from H7 turned out to be too slow for large n.

Invent a method to generate such a word in time $O(n^2)$ and implement it. Do not forget that just adding two n-bit numbers takes time $\Theta(n)$.