RWTH Aachen | **WS 2017**
Theoretical Computer Science | Exercise 9
Rossmanith–Dreier–Kuinke | 9.1.2018

Exercise for Analysis of Algorithms

Exercise T20

In this exercise we consider the following (regular) CFG G:

$$S \to abA \mid bS \mid a$$
$$A \to bA \mid aS$$

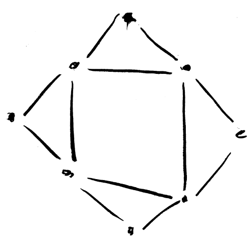
1. Find a generating function for number of words s_n in $L(G)$ that have length n.
2. What is the dominant singularity and what kind of singularity is it?
3. What is the exponential growth of s_n?
4. How precisely can you estimate s_n with just the knowledge of the dominating singularity and its nature?
5. Find a closed formula for s_n with an additive error of at most $O(0.8^n)$.

Exercise T21

An algorithm I computes an optimal independent set for an undirected graph $G = (V, E)$ of size n as follows: It picks a vertex v with maximal degree. If this degree is at most two, then the graph is a collection of cycles and paths and the solution is computed in linear time.

Otherwise, the optimal independent set either contains v (and then cannot contain any vertex in $N(v)$) or it does not. Hence, the algorithm recursively computes the two independent sets $I(G[V - N(v)])$ and $I(G[V - \{v\}])$ and then chooses the bigger one, or the first if they have the same size.

1. Simulate the algorithm on this graph:



2. Estimate its asymptotic running time up to a constant factor.

Exercise H14

In this exercise we will look at 2-3-trees. They are rooted, ordered trees. Each internal node has either two or three children. As usual, the size of a 2-3-tree will be the number of its internal nodes.

1. How can you define 2-3-trees recursively?
2. Enumerate all 2-3-trees of size two. How many are there? How many trees exist of sizes zero and one?
3. Find a generating function $Q(z)$ for the number q_n of 2-3-trees with size n.
4. What is the dominant singularity of $Q(z)$ and what is the exponential growth of q_n? Use a computer algebra system. Do not give up when you see horrifying formulas.