RWTH Aachen | **WS 2017**
Theoretical Computer Science | Exercise 8
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Exercise for Analysis of Algorithms

Exercise T16

Find the exponential growth of the following functions:

a) $2^n n^3$

b) $\left(\frac{2}{3}\right)^{2n} + 5$

c) $[z^n]\frac{1}{\sqrt{1-5z}}$

d) $[z^n]\frac{z^2-1}{(z-1)(z-5)}$

e) $[z^n]\frac{1}{e-e^{3/2-z^2}}$

Exercise T17

Find very large and very small functions with exponential growth 1, 0 and ∞.

Exercise T18

Sort the following generating functions *within one minute* by their exponential growth!

1. $A(z) = \frac{1}{\sqrt{1-z/2}}$
2. $B(z) = \frac{1}{1-e^{z-1/3}}$
3. $C(z) = \frac{(1+z)}{(1-z)}$

Exercise T19

Santa Claus wants to build a new landing strip for his reindeer. He has the following 1×2 tile he can use and rotate by 90 degrees: ▯ . He wants to pave a strip of $2 \times n$. Since his contractor delivers the tiles in a specific color pattern he does not know how he can pave this strip so that it looks best. He wrote a computer program that enumerates all possible ways to pave the strip and then assigns it a beauty-value based on an evaluation function. Afterwards it outputs the best looking option. The evaluation takes $O(n)$ time. He thinks that, if the exponential growth of the running time is less than 4.5^n he can find the most beautiful landing strip in time for Christmas. Can he find it or is Christmas doomed?

Exercise H12

Rudolph contracted another company for delivering the tiles, as Santa had originally in mind (see T19), and they delivered an additional 1×1 tile: ▫ . This increases the options and ultimately the running time for Santa's program. Can he still find the best looking option in time if he also considers the new tile?

Exercise H13

In the lecture the following recurrence was given

$$a_{n+2} - (n+2)a_{n+1} + na_n = n$$

What is the dimension of the solution space? Give a closed form expression for the starting conditions $a_0 = a_1 = 1$.