

Exercise for Analysis of Algorithms

Exercise T9

Let $x \in \mathbf{R}_{\geq 0}$. Is $\lceil \sqrt{x} \rceil = \left\lceil \sqrt{\lceil x \rceil} \right\rceil$?

Exercise T10

How often is the loop in the following excerpt executed if $0 < i$ holds at the beginning?

```
while i <= j
  i := i+j;
  j:=j+10;
```

Exercise T11

Consider the following algorithm that searches an element x in a sorted array a of length $n = km + 1$:

```
i:= 1;
while a[i]<=x
  if a[i]=x then return i;
  i:=i+m;
  if i>n return 0;
for j=i-1 downto max(1,i-(m-1))
  if a[j]=x then return j;
  if a[j]<x then return 0;
return 0;
```

- Draw the search tree and compute the internal and external path length for $n = 10$ and $m = 3$.
- Determine C^+ and C^- for arbitrary m, k .
- What is, for given n , the best choice for m w.r.t. the running time?

Exercise H7

Use summation factors to solve the following recurrence:

$$\begin{aligned} a_0 &= 0 \\ a_n &= \frac{a_{n-1}}{n} + \frac{1}{(n-1)!} \quad \text{for } n \geq 1 \end{aligned}$$

Exercise H8

Use the repertoire method to find a closed form for the following recurrence:

$$\begin{aligned} a_0 &= 5 \\ a_1 &= 9 \\ a_n &= na_{n-1} + n^2a_{n-2} - n^4 - 3n^2 + 5 \quad \text{for } n \geq 2 \end{aligned}$$