

Analysis of Algorithms

Problem 5-1

Solve the following recurrence: $na_n = (n + 1)a_{n-1} + 2n$ for $n \geq 1$ and $a_0 = 0$.

Problem 5-2

Compute the number of iterations of the `while`-loop for $0 < i$ and arbitrary j .

```
while ( i <= j ) {  
    i = i + j;  
    if ( i > j ) then j = j + 10;  
}
```

Problem 5-2

Given an array a of length n , an algorithm compares all pairs $(a[i], a[j])$ for all $i < j \leq n$, and then calls itself recursively on all proper prefixes of a .

How often does the algorithm compare two pairs? Use the repertoire method!

Homework Assignment 5-1 (10 Points)

Use summation factors to solve the following recurrence:

$$\begin{aligned} a_0 &= 0 \\ a_n &= \frac{a_{n-1}}{n} + \frac{1}{(n-1)!} \quad \text{for } n \geq 1 \end{aligned}$$

Homework Assignment 5-2 (10 Points)

Use the repertoire method to find a closed form for the following recurrence:

$$\begin{aligned} a_0 &= 5 \\ a_1 &= 9 \\ a_n &= na_{n-1} + n^2 a_{n-2} - n^4 - 3n^2 + 5 \quad \text{for } n \geq 2 \end{aligned}$$