

## Analysis of Algorithms

### Problem 3-1

In this exercise, we consider Prim's Algorithm, which computes a minimum spanning tree. The input to this algorithm is a graph  $G = (V, E)$ , a weight function on the edges  $w : E \rightarrow \mathbf{R}$  and a starting node  $r$ .

```
1  for each  $u \in V$  do
2       $key[u] \leftarrow \infty$ 
3       $\pi[u] \leftarrow \text{NIL}$ 
4   $key[r] \leftarrow 0$ 
5   $M \leftarrow V$ 
6  while ( $M \neq \emptyset$ ) do
7       $u \leftarrow \text{min-from}(M)$ 
8      for each  $v \in \text{neighbors}(u)$  do
9          if ( $v \in M \wedge (w(u, v) < key[v])$ ) then
10              $\pi[v] \leftarrow u$ 
11              $key[v] \leftarrow w(u, v)$ 
```

Construct the control flow graph, a spanning tree in the control flow graph, the fundamental cycles, a corresponding linear system of equations and a solution to this system.

**Exercise 3-2** The following program prints a lot of numbers. How many?

```
void snapl(int n) {
    if ( n == 0 || n == 2 ) return;
    if ( n == 1 ) std::cout << 42 << std::endl;
    else {
        snapl( n - 2 );
        snapl( n - 3 );
        snapl( n - 2 );
        snapl( n - 3 );
        snapl( n - 2 );
    }
}
```

### Homework Assignment 3-1 (10 Points)

Consider the following program: The input to this program is an array  $a[0, \dots, n - 1]$  that contains  $n$  pairwise distinct integer keys in random order.

- Explain how this program sorts the given array.
- Analyse how often each instruction of the program is executed on average depending on  $n$ .

```

int sel_sort ( int a[], int n ) {
    for ( int i = 0; i < n; ++i ) {
        int min = i;
        for ( int j = i; j < n; ++j ) {
            if ( a[j] < a[min] ) {
                min = j;
            }
        }
        int temp = a[i];
        a[i] = a[min];
        a[min] = temp;
    }
}

```

c) There is only one instruction whose analysis is not trivial. Which one is it?

Make a table for small values of  $n$  by hand that lists the results for this instruction. Compare the table entries with the results from your closed formula that you obtained in b).

### Homework Assignment 3-2 (10 Points)

Solve the following recurrence: Let  $b_1 = b_2 = b_3 = 1$  and

$$b_n = 3b_{n-1} - 4b_{n-2} + 12b_{n-3} \text{ for } n > 3.$$