RWTH Aachen **SS 2012**
Lehrgebiet Theoretische Informatik Tutorial 10
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Analysis of Algorithms — Tutorial

Problem 10-1
Find the generating function for the series given by the following recurrence relation:

$$f_n = f_{n-1} + 2f_{n-2} + 3f_{n-3} + \cdots + nf_0 \text{ for } n > 0 \text{ and } f_0 = 1.$$

Problem 10-2
A software engineer is writing a program that needs at some point a random permutation without fixpoints of the numbers $1, \ldots, n$. A permutation π has a fixpoint k if $\pi(k) = k$. To be more precise: The program has to choose one permutation among all permutations without fixpoints with equal probability.

She wants to reuse as much software as possible and finds a library routine that provides random permutations. Her plan is to call that routine until it delivers a fixpoint free permutation. She worries a bit, however, how many calls she has to make on average.

a) If $\pi \in S_n$ is a random permutation, what is the expected number of fixpoints? Why is the answer not very helpful to her task?

b) Let q_n be the number of fixpoint-free permutations in S_n. Find the EGF for q_n. Start with a recurrence and use the algebraic rules for EGFs.

c) Find the first values of q_n with the help of a computer algebra system.

Homework Assignment 10-1 (10 points)
Expand the generating function from Problem 10-1 in order to find a closed formula for f_n.

Homework Assignment 10-2 (10 Points)
We call a sequence of **push** und **pop** operations ($\uparrow$ and $\downarrow$) *valid*, if it contains the same number of $\uparrow$ and $\downarrow$ and no prefix of the sequence consists of less $\uparrow$ than $\downarrow$. For example, $(\uparrow, \uparrow, \downarrow, \downarrow, \uparrow, \downarrow)$ is valid, while $(\downarrow, \downarrow, \uparrow, \uparrow)$ and $(\uparrow, \downarrow, \downarrow, \uparrow)$ are not valid. The number of $\uparrow$s in a valid sequence is called the *length* of the sequence.
How many valid sequences of length n do exist?