Pattern Languages

Seminar Algorithmic Learning Theory, SS 2015

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What are pattern languages?

- Type of formal languages
- Introduced by Dana Angluin in 1980

Why do they interest us?

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Finding Patterns Common to a Set of Strings

- Pattern languages
  - can be learned from positive data
  - are a natural model for inductive inference

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- Many more substitutions possible!

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- The language of a pattern is the set of all strings of constants we get through substitutions

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- The learner outputs hypotheses after receiving a string
- The learner learns the language, if, after some finite amount of time, the hypotheses are correct and remain the same

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Assuming a learner is presented with a text $s_1, s_2, s_3, \ldots$ of some pattern language

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Assuming there exists an algorithm to find descriptive patterns

- Then paper by Angluin shows:
  Pattern languages can be learned in the limit from positive data

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The membership problem for pattern languages is NP-complete

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- We may still solve this efficiently in special cases!

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Language	Membership	Emptiness	Equivalence	Inclusion
Context-sens.	D	U	U	U
Context-free	D	D	U	U
Regular	D	D	D	D
Pattern lang.	D	D	D	U

Table: D=decidable, U=undecidable

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3. Construct automata that recognize exactly these patterns
4. Finally, select a specific automaton that recognizes descriptive one-variable patterns

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- We can bound $|F| = \mathcal{O}(l^2 \log l)$ where l is the length of the shortest string in S

Anguin's algorithm for finding descriptive one-variable patterns

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Lemma (6.3)

Any pattern accepted by an automaton built from a triple that maximizes $i + j$ is descriptive of S among one variable patterns

Example

Let $S = \{s_1, s_2, s_3\}$ a sample with

$$s_1 = 1101011, s_2 = 10011, s_3 = 11111$$

- We construct F through enumeration
  We get:
  $F = \{(1,1,k),(1,2,k),(2,1,k),(3,1,k),(3,2,k),(4,1,k)\}$
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- In this example we do this for: $(3, 2, 2) \in F$

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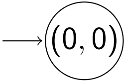

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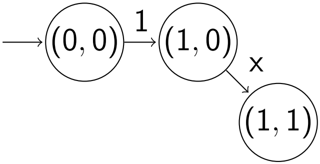

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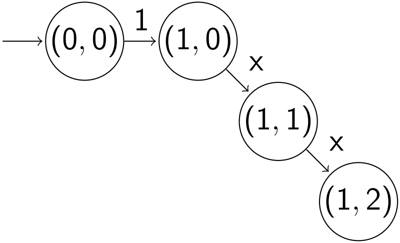

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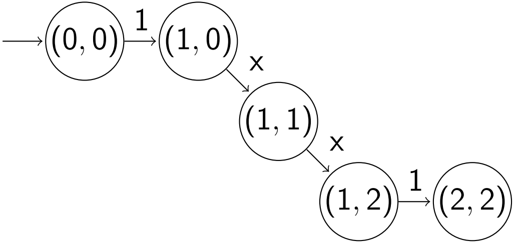

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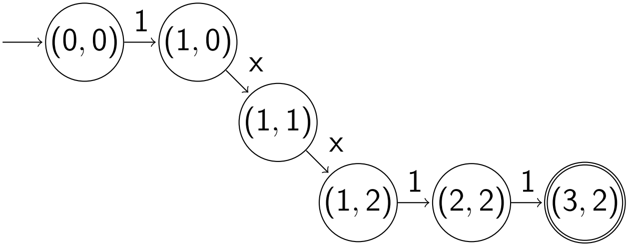

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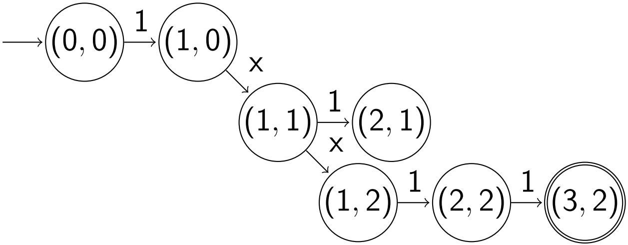

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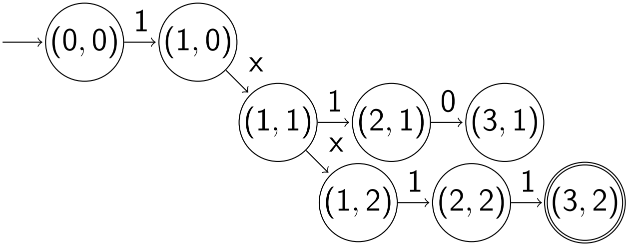

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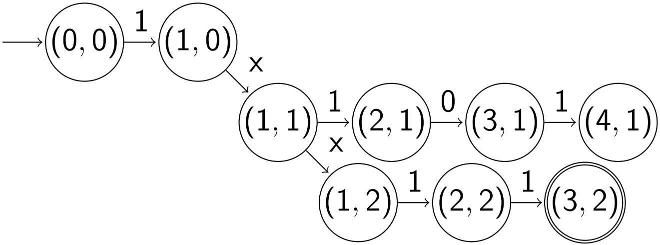

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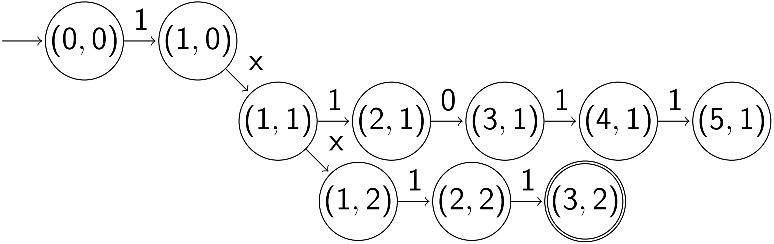

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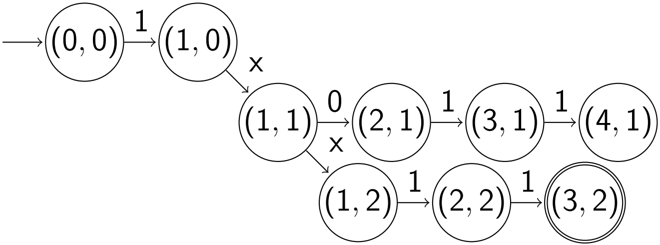

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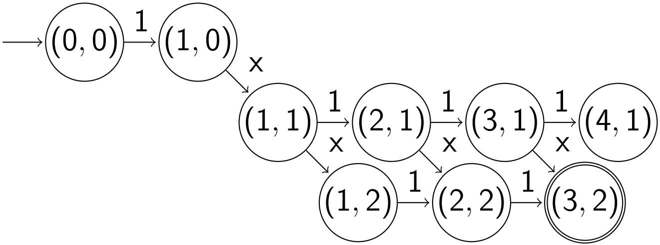
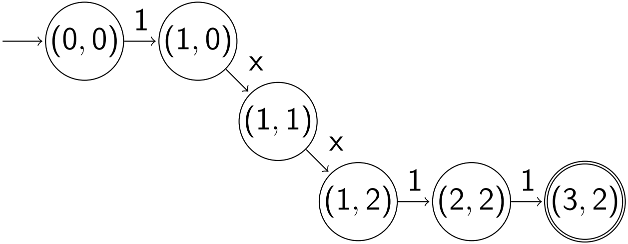


Intersection of all three automata:



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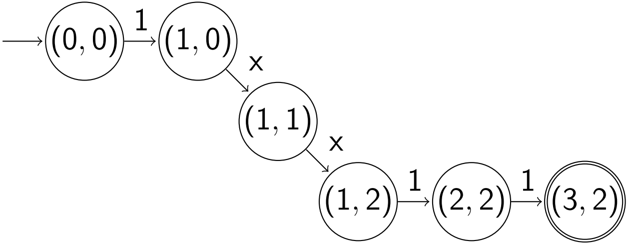

Clearly the automaton recognizes the language $\{1xx11\}$

Recall:
$S = \{s_1, s_2, s_3\}$ with $s_1 = 1101011, s_2 = 10011, s_3 = 11111,$
$F = \{(1, 1, k), (1, 2, k), (2, 1, k), (3, 1, k), (3, 2, k), (4, 1, k)\},$
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- Thus, $1xx11$ is descriptive of S among one-variable patterns

Summary

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- Construct F by enumerating all feasible triples

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- We can bound the number of feasible triples and construct the automata in time polynomial in their sizes
- The algorithm runs in time polynomial in the length of the input

What if we allowed wrong results?

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- Each hypothesis is found in polynomial time

Lange and Wiehagen's algorithm

Idea:

- Only look at strings of minimal length (discard the others)

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- Algorithm will sometimes output wrong hypotheses

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- Polynomial *update* time does not guarantee good *learning* time
- One variable patterns can be learned very efficiently - will be covered in next talk!

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- Much work done in related fields!

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- Pattern languages: model for inductive inference
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- Special case: polynomial-time algorithm for one-variable patterns
- Lange/Wiehagen algorithm: inconsistent algorithm turns out to be very effective

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