Parameterized Algorithms WS 2021
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Exercise Sheet 09

In this exercise sheet we take a look at the *maximum internal spanning tree problem*. The problems asks at most how many internal vertices a spanning tree of a given graph G can contain. We consider the paramterized problem p-IST that is parameterized by the number of internal vertices k.

Task T28

Find a polynomnial time algorithm that finds in a graph G either a spanning tree with k internal vertices or an independent set of size $2n/3$ for $n > 3k$.

Task T29

Find a kernel of size $3k$ for p-IST. Use the result of Exercise T28 and the following lemma:

Lemma 1. *If $n \geq 3$, and I is an independent set of G of cardinality at least $2n/3$, then there are nonempty subsets $S \subseteq V \setminus I$ and $L \subseteq I$ such that*

1. $N(L) = S$,
2. *$B(L, S)$ has a spanning tree such that all vertices fo S and $|S| - 1$ vertices of L are internal.*

Moreover, given a graph on at least 3 vertices and an independent set of cardinality at least $2n/3$, such subsets can be found in time polynomial in the size of G.

The bipartite graph $B(S, L)$ describes the graph induced by G on $S \cup L$ without edges between vertices of S or between vertices of L.

Task H19 (15pts)

It seems that we overlooked a detail in Exercise T29. To fix it, you have to prove the following lemma:

Lemma 2. *If G has a spanning tree with k internal vertices, then G has a spanning tree with at least k internal vertices which all the vertices of S and exactly $|S|-1$ vertices of L are internal.*

Is this enough?

Task H20 (5pts)

Use the results above to find a parameterized algorithm that solves p-IST in time $8^k n^{O(1)}$.