RWTH AACHEN UNIVERSITY

Date: November 22nd, 2021

Exercise Sheet 05

Task T16

A graph $G = (V, E)$ is a *split graph* if its vertex set can be partitioned into sets C and I such that C is a clique and I is an independent set. Show that a graph is a split graph if and only if it does not contain the following three graphs as induced subgraphs: TODO $C_5, C_4, 2P_2$.

In the *Split Vertex Deletion* problem, given a graph G and an integer k, the task is to check if one can delete at most k vertices from G to obtain a split graph. Find an fpt algorithm to solve this problem. Can you find a different algorithm which solves this problem in $2^k n^{\mathcal{O}(1)}$?

Task T17

The r-Regular Vertex Deletion problem is defined as follows: given a graph G and an integer k, decide whether there is a set $S \subseteq V(G)$ of size at most k whose deletion results in an r-regular graph. A graph is r-regular if every vertex has degree exactly r. Show that this problem admits an algorithm with running time $O((r+2)^k \cdot \mathrm{poly}(n))$.

Task H11 (5 credits)

Let $G = (L \dot\cup R, E)$ be a bipartite graph. Suppose that $L_1 \dot\cup L_2 = L$ and $R_1 \dot\cup R_2 = R$ are partitions of the vertex sets L and R. Prove the following:

1. $(L_1 \cup R_1, L_2 \cup R_2, E)$ is a bipartite graph iff there are no paths for the following pairs of vertex sets: L_1 and L_2; L_2 and R_2; R_2 and R_1; R_1 and L_1.
2. One can find a minimum set X such that $G - X$ does not contain any of the above paths in polynomial time [Hint: use a flow algorithm].

Task H12 (5 credits)

Use the insights you gained from H11 to design a $O(3^k n^{O(1)})$-algorithm for Odd Cycle Transversal using iterative compression.