

Exercise Sheet 11

Task T33

Given a binary matrix of size $m \times n$, the k -rows problem consists in finding k rows such that their conjunction is 0, i.e., a row with n zeros.

If the k -rows problem is FPT on matrices of size $\frac{n}{2} \times n$, is the k -rows problem FPT on square matrices of (size $n \times n$)? Could you use an FPT algorithm for k -rows on $\frac{n}{2} \times n$ matrices to solve the problem on matrices of size $n \times n$?

Task T34

A graph H is a d -cluster graph H if H has d connected components and every connected component of H is a clique.

For a set of *adjacencies* $A \subseteq \binom{V(G)}{2}$, we denote with $G \oplus A$ the graph $(V(G), E(G) \Delta A)$. Show that the d -CLUSTERING is in FPT with a subexponential funktion in k :

- Input: A graph $G = (V, E)$
- Parameter: k
- Question: Can you add or remove edges at most k edges in G such that the resulting graph is a d -cluster graph? That is, find an A of size at most k such that $G \oplus A$ is a d -cluster graph.

Lemma 1. *If the vertices of a simple graph G with k edges are colored independently and uniformly at random with $\lceil \sqrt{8k} \rceil$ colors, then the probability that $E(G)$ is properly colored is at least $2^{-\sqrt{k/2}}$.*