

Exercise Sheet 10

Task T30

The MSO type of a structure S with a finite domain is the set of all MSO formulas ϕ with $S \models \phi$. Let us say that the q -type are the formulas in the type that have at most q variables. For simplicity we always assume that formulas are in prenex normal form.

- Is the q -type of a structure finite or can it be infinite?
- If it is infinite, are there only finitely many equivalence classes with regard to logical equivalence between formulas?
- How could representatives of these equivalence classes look like?

Task T31

For a graph $G = (V, E)$ with a t -protrusion X we look at the structure $S = (V, E, X, y_1, \dots, y_r)$ where y_1, \dots, y_r is the border of X .

Show that the following problem is in FPT for some function f :

- Input: $S = (V, E, X, y_1, \dots, y_r)$, $t, q \in \mathbb{N}$
- Parameter: t, q
- Question: If X' is another t -protrusion with the same border as X we define $S' = (V', E', X', y_1, \dots, y_r)$. Is there a smaller X' with $S \models \phi$ iff $S' \models \phi$ for all MSO-formulas ϕ with at most q variables?

Task T32

Prove that VERTEX COVER has finite integer index and k -PATH does not.

Task H21 (10pts)

Let t be a constant. Design an efficient algorithm that solves the following problem in polynomial time:

- Input: A graph G and a number k
- Output: A t -protrusion in G of size at least k or the answer that no such protrusion exists.

The degree of a polynomial that upper bounds the running time may depend on t .

Task H22 (10pts)

Find a graph class that excludes some H as a topological minor, but contains *every* graph H as a minor (i.e., contains a graph that has H as a minor).