Parameterized Algorithms WS 2021
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Exercise Sheet 01

Task T1

The Independent Set problem is defined as follows. Given a graph $G = (V, E)$ and an integer k, is ther a set S of size k such that for all $u, v \in S$ where $u \neq v$ it holds $uv \notin E(G)$? Is Independent Set restricted to graphs of maximal degree d, where d is a constant, fixed paramater tractable parameterized by the size of the solution k?

Task T2

The Triangle Vertex Deletion problem is defined as follows. Given a graph $G = (V, E)$ and an integer parameter k, are there k vertices whose deletion results in a graph with no cycles of length three? Show that this problem is fixed-parameter tractable. What is the running time of your algorithm? Is there some easy way to improve the running time?

Task T3

Given a boolean formula φ in CNF with n variables and m clauses and an integer k, you have to decide whether there exists an assignment to the variables that satisfies at least k clauses. Assume that the literals appearing in a clause are all distinct and that no clause contains a literal and its negation. We first consider several special cases.

1. If φ contains only unit clauses (clauses with one literal), then how can you find the optimum assignment?
2. If there are k clauses in φ that each contain k literals, then show that one can find an assignment satisfying all these clauses in $|\varphi|$ time.
3. Show that one can always find an assignment that satisfies at least $m/2$ clauses of φ.

Use these facts to design an FPT-algorithm with k as parameter.

Task H1 (5 credits)

Use the algorithm presented in the lecture to solve the following instance of Center String. Recall that the parameter is the hamming distance which we fix to be $d = 2$.

$$\begin{aligned} s_1 &= bbbb \\ s_2 &= aabb \\ s_3 &= aaaa \\ s_4 &= abaa \end{aligned}$$

Task H2 (10 credits)

a) Invent a reduction rule for VERTEX COVER that removes all pendant vertices (vertices of degree one). It should generate an equivalent instance in polynomial time.

b) Design a bounded search tree algorithm for VERTEX COVER in graphs of minimum degree two. Analyze its running time. It should be faster than $1.7^k n^{O(1)}$.

c) Use the results above to get an algorithm that solves VERTEX COVER on general graphs with the running time of b).