RWTH Aachen **WS 2018**
Lehrgebiet Theoretische Informatik Problem Set 5
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Parameterized Algorithms Tutorial

Tutorial Exercise T1

Let G be a graph and let $S \subseteq V(G)$ be some vertex subset. Show that the following properties are MSO-expressible:

- S is a vertex cover of G
- S is an independent set of G
- G is a connected graph
- S induces a cycle in G
- G has a Hamiltonian path
- S induces an even cycle in G

Tutorial Exercise T2

The problem 3-Colorability is defined as follows: Given a graph G decide if it is possible to assign every node of G one of three colors, such that no two nodes with the same color are adjacent. This problem is fpt parameterized by the treewidth of the graph. Give an algorithm that solves the problem given a tree decomposition of width w in time $O(3^w \cdot w \cdot n)$.

Tutorial Exercise T3

Let k be a constant and consider the class of graphs that have a vertex cover of size *exactly* k. Does this class define a hereditary property? What can you say about the class of graphs that have a vertex cover of size *at most* k? In case you believe that the property is hereditary, how large is the forbidden set?

Homework H1

Let G be a graph and let $S \subseteq V(G)$ be some vertex subset. Show that the following properties are MSO-expressible:

- S is a dominating set of G
- S is a distance-2 dominating set of G
- G induces a longest path in G
- S induces a steiner tree in G

Homework H2

The problem Dominating Set is defined as follows: Given a graph G find the smallest set $S \subseteq V(G)$ such that every node of G is either in S or has a neighbor in S. This problem is fpt parameterized by the treewidth of the graph. Give an algorithm that solves the problem given a tree decomposition of width w in time $O(9^w \cdot w \cdot n)$.

Homework H3

Let Π be a hereditary property that *excludes* some clique and some independent set. Show that Π-Induced Subgraph is FPT.