Graph Properties

Definition

A graph property Π is a class of graphs that is closed under graph isomorphisms.

That is, if two graphs G_1 and G_2 are isomorphic, both belong to Π or both don't.

Graph Properties

Example

- Connected graphs
- Trees
- Graphs containing a clique of size 100
- Planar graphs
- Regular graphs
- Finite graphs

Graph Properties

These are not graph properties:

- Graphs whose nodes are natural numbers
- Every nonemtpy finite set of graphs
- (For Logicians: Each set of graphs)

Hereditary Graph Properties

A graph property Π is called hereditary if the following holds:

Let $G \in \Pi$ and H be an induced subgraph of G.

Then $H \in \Pi$ as well.

In other words: Π is closed under taking induced subgraphs.

Hereditary Graph Properties

A graph property Π is called hereditary if the following holds:

Let $G \in \Pi$ and H be an induced subgraph of G.

Then $H \in \Pi$ as well.

In other wor

Questions:

1. Does the empty graph belong to every hereditary graph property?
2. Are graph properties lattices with respect to the induced subgraph relation?

Hereditary Graph Properties

Which graph properties are hereditary?

- Bipartite graphs
- Complete graphs
- Planar graphs
- Trees
- Connected graphs
- Graphs of diameter at most d
- Regular graphs

Hereditary Graph Properties

Which graph properties are hereditary?

- Forests
- Graphs containing an independent set of size 8
- Graphs with at least 17 nodes
- Graphs containing no matching of size 35
- 5-regular graphs
- Infinite graphs
- Chordal graphs

Characterization by Obstruction Sets

Definition

A graph property Π has a characterization by obstruction sets if there is a graph property $\mathcal{F}$ such that $G \in \Pi$ if and only if $\mathcal{F}$ does not contain an induced subgraph of G.

Characterization by Obstruction Sets

Definition

A graph property Π has a chara … if
there is a graph property $\mathcal{F}$ suc … does
not contain an induced subgrap …

Question:

Does every hereditary graph property have a characterization by obstruction sets?

Characterization by Obstruction Sets

Definition

A graph property Π has a chara… if
there is a graph property $\mathcal{F}$ suc… does
not contain an induced subgrap…

Question:

Does every hereditary graph…

Answer:

Yes. Choose $\mathcal{F} = \mathcal{G} - \Pi$ with $\mathcal{G}$ containing all graphs.

Finite Obstruction Sets

Definition

A graph property Π has a finite characterization by obstruction sets if it has a characterization by $\mathcal{F}$, and $\mathcal{F}$ contains only a finite number of non-isomorphic graphs.

Finite Obstruction Sets

Which graph properties have a finite characterization by obstruction sets?

- Graphs containing an independent set of size 7?
- Bipartite graphs?
- Forests?
- Planar graphs?
- 5-colorable graphs?
- Graphs containing a vertex cover of size k?

Finite Obstruction Sets

Which graph properties have a finite characterization by obstruction sets?

- Triangle-free graphs?
- Graphs without any k-cliques?
- Graphs of diameter at most d?
- Cycle-free graphs?
- Graphs not containing any cycle of length k?