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Problem Set 10
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Parameterized Algorithms Tutorial

Tutorial Exercise T26

Let $G = (L \uplus R, E)$ be a bipartite graph. Suppose that $L_1 \uplus L_2 = L$ and $R_1 \uplus R_2 = R$ are partitions of the vertex sets L and R. Prove the following:

1. $(L_1 \cup R_1, L_2 \cup R_2, E)$ is a bipartite graph iff there are no paths for the following pairs of vertex sets: L_1 and L_2; L_2 and R_2; R_2 and R_1; R_1 and L_1.
2. One can find a minimum set X such that $G - X$ does not contain any of the above paths in polynomial time [Hint: use a flow algorithm].

Tutorial Exercise T27

Use the insights you gained from T26 to design a $O(3^k n^{O(1)})$-algorithm for Odd Cycle Transversal using iterative compression.

Homework H22

Given a graph $G = (V, E)$, a *perfect code* for G is a vertex set $S \subseteq V(G)$ such that for all $v \in V(G)$ there is exactly one vertex in $N[v] \cap S$. The Perfect Code problem is defined as follows: given a graph $G = (V, E)$ and an integer parameter k, decide whether G has a perfect code with k vertices. This problem is W[1]-complete on general graphs. Show that this problem is fixed-parameter tractable if we assume that the input graph is planar. Use the fact that every planar graph has a vertex of degree at most five.

Homework H23

The r-Regular Vertex Deletion problem is defined as follows: given a graph G and an integer k, decide whether there is a set $S \subseteq V(G)$ of size at most k whose deletion results in an r-regular graph. A graph is r-regular if every vertex has degree exactly r. Show that this problem admits an algorithm with running time $O((r+2)^k \cdot \text{poly}(n))$.