RWTH Aachen | **WS 2011/12**
Lehrgebiet Theoretische Informatik | Problem Set 2
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Parameterized Algorithms Tutorial

Tutorial Exercise T4

The Nemhauser–Trotter-Theorem states that given an undirected graph $G = (V, E)$, one can in polynomial time to find two disjoint vertex sets $C_0 \uplus V_0 \subseteq V$ such that

1. If $D \subseteq V_0$ is a vertex cover of $G[V_0]$, then $D \cup C_0$ is a vertex cover of G.
2. There is an optimal vertex cover S of G with $C_0 \subseteq S$.
3. Every vertex cover of $G[V_0]$ has size at least $|V_0|/2$.

Show that:

1. If S' is an optimal vertex cover of $G[V_0]$, then $S' \cup C_0$ is an optimal vertex cover of G.
2. If G has a vertex cover of size k, then $|V_0| + |C_0| \leq 2k$.

Tutorial Exercise T5

Let Π be a decidable parameterized problem. Show that Π is fixed-parameter tractable if and only if there exists a kernelization algorithm for it.

Tutorial Exercise T6

Consider the Planar Independent Set problem: Given a planar graph G and an integer k, decide whether G has an independent set of size at least k. Design an algorithm that takes as input (G, k), where G is planar, and outputs in polynomial time an equivalent instance (G', k') such that $|V(G')| = O(k)$.

Homework H3

Consider the $n \times n$ sliding puzzle which consists of a frame with $n^2 - 1$ square tiles with numbers on them; the frame has one missing tile and this enables the others to move horizontally and vertically. See, for instance, the following 4×4 puzzle with numbers in hexadecimal:

1	3	6	4
5		7	8
9	2	F	B
D	A	E	C

The puzzle is *solved*, if all numbers are sorted both row-, and column-wise, with the smallest number appearing in the top left corner of the frame, and the empty space on the bottom right corner.

The SLIDING PUZZLE problem is defined as follows: given an $n \times n$ sliding puzzle and an integer k, decide whether one can solve the puzzle in at most k moves. A move consists in moving a tile either horizontally or vertically one step.

- Show that this problem is in FPT by giving a bounded search-tree algorithm for it.
- Give an algorithm that constructs a kernel of polynomial size in polynomial time.

Homework H4

Given a graph $G = (V, E)$, an *induced matching* of G is a set of edges $F \subseteq E$, such that the edge set of the induced subgraph $G[V(F)]$ is F itself. The *size* of an induced matching is the number of edges in it. The INDUCED MATCHING problem is given a graph G and an integer k, to decide whether G has an induced matching of size at least k.

Design a linear kernel for this problem on graphs of maximum degree d, where d is a constant.