RWTH Aachen **WS 2011/12**
Lehrgebiet Theoretische Informatik Problem Set 1
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Parameterized Algorithms Tutorial

Tutorial Exercise T1

A question from the lecture concerns the following algorithm for the Minimum Vertex Cover problem: choose an arbitrary edge $e = \{u, v\}$ that has not yet been covered and branch on the two subcases; on one branch include u in the solution and in the other include v in the solution. Return the smaller of the two solutions.
Does this algorithm run in FPT-time if parameterized by the size of the minimal vertex cover? If yes, provide a formal proof. If no, provide a generic counterexample.

Tutorial Exercise T2

Design a parameterized algorithm for the Independent Set problem on planar graphs. Why does your approach not work in the case of general graphs?

Tutorial Exercise T3

Use the algorithm presented in the lecture to solve the following instance of Center String. Recall that the parameter is the hamming distance which we fix to be $d = 2$.

$$\begin{aligned} s_1 &= bbbb \\ s_2 &= aabb \\ s_3 &= aaaa \\ s_4 &= abaa \end{aligned}$$

Homework H1

The Cluster Vertex Deletion problem is defined as follows: given a graph $G = (V, E)$ and an integer parameter k, does there exist a set S of size at most k such that $G[V \setminus S]$ consists of a collection of disjoint cliques. The cliques are disjoint in the sense that they do not share vertices and/or edges and there is no edge with one endpoint in one clique and the other in a different clique. Design an algorithm that runs in FPT-time wrt k as parameter. [Hint: A graph is a cluster graph if and only if it does not contain a path with three vertices as an induced subgraph.]

Homework H2

An input to the Hitting Set problem consists of a finite universe U and a set family $\mathcal{F} \subseteq 2^U$. The question is to decide whether there exists a set $H \subseteq U$ of size at most k such that $H \cap S \neq \emptyset$ for all $S \in \mathcal{F}$, that is, the set H contains at least one element from each set of the family $\mathcal{F}$.
A variant of this problem, known as 3-Hitting Set, imposes the additional restriction that $|S| \leq 3$ for all $S \in \mathcal{F}$. Design an algorithm for this variant that runs in FPT-time where k is the parameter.